

## Computation Laws For Classes

### Commutativity of Union and Intersection

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**Proposition 0.1.** Let  $A, B$  be classes. Then  $A \cup B = B \cup A$ .

*Proof.* Let us show that  $A \cup B \subseteq B \cup A$ . Let  $x \in A \cup B$ . Then  $x \in A$  or  $x \in B$ . Hence  $x \in B$  or  $x \in A$ . Thus  $x \in B \cup A$ . End.

Let us show that  $B \cup A \subseteq A \cup B$ . Let  $x \in B \cup A$ . Then  $x \in B$  or  $x \in A$ . Hence  $x \in A$  or  $x \in B$ . Thus  $x \in A \cup B$ . End.  $\square$

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**Proposition 0.2.** Let  $A, B$  be classes. Then  $A \cap B = B \cap A$ .

*Proof.* Let us show that  $A \cap B \subseteq B \cap A$ . Let  $x \in A \cap B$ . Then  $x \in A$  and  $x \in B$ . Hence  $x \in B$  and  $x \in A$ . Thus  $x \in B \cap A$ . End.

Let us show that  $B \cap A \subseteq A \cap B$ . Let  $x \in B \cap A$ . Then  $x \in B$  and  $x \in A$ . Hence  $x \in A$  and  $x \in B$ . Thus  $x \in A \cap B$ . End.  $\square$

### Associativity of Union and Intersection

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**Proposition 0.3.** Let  $A, B, C$  be classes. Then  $(A \cup B) \cup C = A \cup (B \cup C)$ .

*Proof.* Let us show that  $((A \cup B) \cup C) \subseteq A \cup (B \cup C)$ . Let  $x \in (A \cup B) \cup C$ . Then  $x \in A \cup B$  or  $x \in C$ . Hence  $x \in A$  or  $x \in B$  or  $x \in C$ . Thus  $x \in A$  or  $x \in (B \cup C)$ . Therefore  $x \in A \cup (B \cup C)$ . End.

Let us show that  $A \cup (B \cup C) \subseteq (A \cup B) \cup C$ . Let  $x \in A \cup (B \cup C)$ . Then  $x \in A$  or  $x \in B \cup C$ . Hence  $x \in A$  or  $x \in B$  or  $x \in C$ . Thus  $x \in A \cup B$  or  $x \in C$ . Therefore  $x \in (A \cup B) \cup C$ . End.  $\square$

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**Proposition 0.4.** Let  $A, B, C$  be classes. Then  $(A \cap B) \cap C = A \cap (B \cap C)$ .

*Proof.* Let us show that  $((A \cap B) \cap C) \subseteq A \cap (B \cap C)$ . Let  $x \in (A \cap B) \cap C$ . Then  $x \in A \cap B$  and  $x \in C$ . Hence  $x \in A$  and  $x \in B$  and  $x \in C$ . Thus  $x \in A$  and  $x \in (B \cap C)$ . Therefore  $x \in A \cap (B \cap C)$ . End.

Let us show that  $A \cap (B \cap C) \subseteq (A \cap B) \cap C$ . Let  $x \in A \cap (B \cap C)$ . Then  $x \in A$  and  $x \in B \cap C$ . Hence  $x \in A$  and  $x \in B$  and  $x \in C$ . Thus  $x \in A \cap B$  and  $x \in C$ . Therefore  $x \in (A \cap B) \cap C$ . End.  $\square$

## Distributivity of Union and Intersection

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**Proposition 0.5.** Let  $A, B, C$  be classes. Then  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ .

*Proof.* Let us show that  $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ . Let  $x \in A \cap (B \cup C)$ . Then  $x \in A$  and  $x \in B \cup C$ . Hence  $x \in A$  and  $(x \in B$  or  $x \in C)$ . Thus  $(x \in A$  and  $x \in B)$  or  $(x \in A$  and  $x \in C)$ . Therefore  $x \in A \cap B$  or  $x \in A \cap C$ . Hence  $x \in (A \cap B) \cup (A \cap C)$ . End.

Let us show that  $((A \cap B) \cup (A \cap C)) \subseteq A \cap (B \cup C)$ . Let  $x \in (A \cap B) \cup (A \cap C)$ . Then  $x \in A \cap B$  or  $x \in A \cap C$ . Hence  $(x \in A$  and  $x \in B)$  or  $(x \in A$  and  $x \in C)$ . Thus  $x \in A$  and  $(x \in B$  or  $x \in C)$ . Therefore  $x \in A$  and  $x \in B \cup C$ . Hence  $x \in A \cap (B \cup C)$ . End.  $\square$

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**Proposition 0.6.** Let  $A, B, C$  be classes. Then  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ .

*Proof.* Let us show that  $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ . Let  $x \in A \cup (B \cap C)$ . Then  $x \in A$  or  $x \in B \cap C$ . Hence  $x \in A$  or  $(x \in B$  and  $x \in C)$ . Thus  $(x \in A$  or  $x \in B)$  and  $(x \in A$  or  $x \in C)$ . Therefore  $x \in A \cup B$  and  $x \in A \cup C$ . Hence  $x \in (A \cup B) \cap (A \cup C)$ . End.

Let us show that  $((A \cup B) \cap (A \cup C)) \subseteq A \cup (B \cap C)$ . Let  $x \in (A \cup B) \cap (A \cup C)$ . Then  $x \in A \cup B$  and  $x \in A \cup C$ . Hence  $(x \in A$  or  $x \in B)$  and  $(x \in A$  or  $x \in C)$ . Thus  $x \in A$  or  $(x \in B$  and  $x \in C)$ . Therefore  $x \in A$  or  $x \in B \cap C$ . Hence  $x \in A \cup (B \cap C)$ . End.  $\square$

## Idempocpy Laws for Union and Intersection

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**Proposition 0.7.** Let  $A$  be a class. Then  $A \cup A = A$ .

*Proof.*  $A \cup A = \{x \mid x \in A \text{ or } x \in A\}$ . Hence  $A \cup A = \{x \mid x \in A\}$ . Thus  $A \cup A = A$ .  $\square$

**Proposition 0.8.** Let  $A$  be a class. Then  $A \cap A = A$ .

*Proof.*  $A \cap A = \{x \mid x \in A \text{ and } x \in A\}$ . Hence  $A \cap A = \{x \mid x \in A\}$ . Thus  $A \cap A = A$ .  $\square$

## Distributivity of Complement

**Proposition 0.9.** Let  $A, B, C$  be classes. Then  $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$ .

*Proof.* Let us show that  $A \setminus (B \cap C) \subseteq (A \setminus B) \cup (A \setminus C)$ . Let  $x \in A \setminus (B \cap C)$ . Then  $x \in A$  and  $x \notin B \cap C$ . Hence it is wrong that  $(x \in B \text{ and } x \in C)$ . Thus  $x \notin B$  or  $x \notin C$ . Therefore  $x \in A$  and  $(x \notin B \text{ or } x \notin C)$ . Then  $(x \in A \text{ and } x \notin B)$  or  $(x \in A \text{ and } x \notin C)$ . Hence  $x \in A \setminus B$  or  $x \in A \setminus C$ . Thus  $x \in (A \setminus B) \cup (A \setminus C)$ . End.

Let us show that  $((A \setminus B) \cup (A \setminus C)) \subseteq A \setminus (B \cap C)$ . Let  $x \in (A \setminus B) \cup (A \setminus C)$ . Then  $x \in A \setminus B$  or  $x \in A \setminus C$ . Hence  $(x \in A \text{ and } x \notin B)$  or  $(x \in A \text{ and } x \notin C)$ . Thus  $x \in A$  and  $(x \notin B \text{ or } x \notin C)$ . Therefore  $x \in A$  and not  $(x \in B \text{ and } x \in C)$ . Then  $x \in A$  and not  $x \in B \cap C$ . Hence  $x \in A \setminus (B \cap C)$ . End.  $\square$

**Proposition 0.10.** Let  $A, B, C$  be classes. Then  $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ .

*Proof.* Let us show that  $A \setminus (B \cup C) \subseteq (A \setminus B) \cap (A \setminus C)$ . Let  $x \in A \setminus (B \cup C)$ . Then  $x \in A$  and  $x \notin B \cup C$ . Hence it is wrong that  $(x \in B \text{ or } x \in C)$ . Thus  $x \notin B$  and  $x \notin C$ . Therefore  $x \in A$  and  $(x \notin B \text{ and } x \notin C)$ . Then  $(x \in A \text{ and } x \notin B)$  and  $(x \in A \text{ and } x \notin C)$ . Hence  $x \in A \setminus B$  and  $x \in A \setminus C$ . Thus  $x \in (A \setminus B) \cap (A \setminus C)$ . End.

Let us show that  $((A \setminus B) \cap (A \setminus C)) \subseteq A \setminus (B \cup C)$ . Let  $x \in (A \setminus B) \cap (A \setminus C)$ . Then  $x \in A \setminus B$  and  $x \in A \setminus C$ . Hence  $(x \in A \text{ and } x \notin B)$  and  $(x \in A \text{ and } x \notin C)$ . Thus  $x \in A$  and  $(x \notin B \text{ and } x \notin C)$ . Therefore  $x \in A$  and not  $(x \in B \text{ or } x \in C)$ . Then  $x \in A$  and not  $x \in B \cup C$ . Hence  $x \in A \setminus (B \cup C)$ . End.  $\square$

## Subclass Laws

**Proposition 0.11.** Let  $A, B$  be classes. Then  $A \subseteq A \cup B$ .

*Proof.* Let  $x \in A$ . Then  $x \in A$  or  $x \in B$ . Hence  $x \in A \cup B$ .  $\square$

**Proposition 0.12.** Let  $A, B$  be classes. Then  $A \cap B \subseteq A$ .

*Proof.* Let  $x \in A \cap B$ . Then  $x \in A$  and  $x \in B$ . Hence  $x \in A$ .  $\square$

**Proposition 0.13.** Let  $A, B$  be classes. Then  $A \subseteq B$  iff  $A \cup B = B$ .

*Proof.* Case  $A \subseteq B$ .

Let us show that  $A \cup B \subseteq B$ . Let  $x \in A \cup B$ . Then  $x \in A$  or  $x \in B$ . If  $x \in A$  then  $x \in B$ . Hence  $x \in B$ . End.

Let us show that  $B \subseteq A \cup B$ . Let  $x \in B$ . Then  $x \in A$  or  $x \in B$ . Hence  $x \in A \cup B$ . End. End.

Case  $A \cup B = B$ . Let  $x \in A$ . Then  $x \in A$  or  $x \in B$ . Hence  $x \in A \cup B = B$ . End.  $\square$

**Proposition 0.14.** Let  $A, B$  be classes. Then  $A \subseteq B$  iff  $A \cap B = A$ .

*Proof.* Case  $A \subseteq B$ .

Let us show that  $A \cap B \subseteq A$ . Let  $x \in A \cap B$ . Then  $x \in A$  and  $x \in B$ . Hence  $x \in A$ . End.

Let us show that  $A \subseteq A \cap B$ . Let  $x \in A$ . Then  $x \in B$ . Hence  $x \in A$  and  $x \in B$ . Thus  $x \in A \cap B$ . End. End.

Case  $A \cap B = A$ . Let  $x \in A$ . Then  $x \in A \cap B$ . Hence  $x \in A$  and  $x \in B$ . Thus  $x \in B$ . End.  $\square$

## Complement Laws

**Proposition 0.15.** Let  $A$  be a class. Then  $A \setminus A = \emptyset$ .

*Proof.*  $A \setminus A$  has no elements. Indeed  $A \setminus A = \{x \mid x \in A \text{ and } x \notin A\}$ .

Hence the thesis.  $\square$

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**Proposition 0.16.** Let  $A$  be a class. Then  $A \setminus \emptyset = A$ .

*Proof.*  $A \setminus \emptyset = \{x \mid x \in A \text{ and } x \notin \emptyset\}$ . No element is an element of  $\emptyset$ . Hence  $A \setminus \emptyset = \{x \mid x \in A\}$ . Then we have the thesis.  $\square$

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**Proposition 0.17.** Let  $A, B$  be classes. Then  $A \setminus (A \setminus B) = A \cap B$ .

*Proof.* Let us show that  $A \setminus (A \setminus B) \subseteq A \cap B$ . Let  $x \in A \setminus (A \setminus B)$ . Then  $x \in A$  and  $x \notin A \setminus B$ . Hence  $x \notin A$  or  $x \in B$ . Thus  $x \in B$ . Therefore  $x \in A \cap B$ . End.

Let us show that  $A \cap B \subseteq A \setminus (A \setminus B)$ . Let  $x \in A \cap B$ . Then  $x \in A$  and  $x \in B$ . Hence  $x \notin A \setminus B$ . Thus  $x \notin A \setminus (A \setminus B)$ . Therefore  $x \in A \setminus (A \setminus B)$ . End.  $\square$

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**Proposition 0.18.** Let  $A, B$  be classes. Then  $B \subseteq A$  iff  $A \setminus (A \setminus B) = B$ .

*Proof.* Case  $B \subseteq A$ . Obvious.

Case  $A \setminus (A \setminus B) = B$ . Then every element of  $B$  is an element of  $A \setminus (A \setminus B)$ . Thus every element of  $B$  is an element of  $A$ . Then we have the thesis. End.  $\square$

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**Proposition 0.19.** Let  $A, B, C$  be classes. Then  $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$ .

*Proof.* Let us show that  $A \cap (B \setminus C) \subseteq (A \cap B) \setminus (A \cap C)$ . Let  $x \in A \cap (B \setminus C)$ . Then  $x \in A$  and  $x \in B \setminus C$ . Hence  $x \in A$  and  $x \in B$ . Thus  $x \in A \cap B$  and  $x \notin C$ . Therefore  $x \notin A \cap C$ . Then we have  $x \in (A \cap B) \setminus (A \cap C)$ . End.

Let us show that  $((A \cap B) \setminus (A \cap C)) \subseteq A \cap (B \setminus C)$ . Let  $x \in (A \cap B) \setminus (A \cap C)$ . Then  $x \in A$  and  $x \in B$ .  $x \notin A \cap C$ . Hence  $x \notin C$ . Thus  $x \in B \setminus C$ . Therefore  $x \in A \cap (B \setminus C)$ . End.  $\square$