Exponentiation of Even and Odd Numbers

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Proposition 0.1. Let n, m be natural numbers such that $m > 0$. Assume that n is even. Then n^m is even.

Proof. Take a natural number k such that $n = 2 \cdot k$. Consider a natural number m' such that $m = m' + 1$. Then $n^m = (2 \cdot k)^m = (2^m \cdot (k^m)) = (2^{m'+1} \cdot (k^m)) = (2^{m'} \cdot 2) \cdot (k^m) = (2 \cdot 2^{m'}) \cdot (k^m) = 2 \cdot (2^{m'} \cdot (k^m))$. Hence n^m is even. $\square$

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Proposition 0.2. Let n, m be natural numbers. Assume that n is odd. Then n^m is odd.

Proof. Define $\Phi = \{m' \in \mathbb{N} \mid n^{m'} \text{ is odd } \}$.

(1) Φ contains 0. Indeed $n^0 = 1$ and 1 is odd.

(2) For all $m' \in \Phi$ we have $m' + 1 \in \Phi$.
Proof. Let $m' \in \Phi$. We have $n^{m'+1} = n^{m'} \cdot n$. $n^{m'}$ is odd. Hence we can take a natural number k such that $n^{m'} = (2 \cdot k) + 1$. Then $n^{m'+1} = ((2 \cdot k) + 1) \cdot n = ((2 \cdot k) \cdot n) + (1 \cdot n) = ((2 \cdot k) \cdot n) + n = (2 \cdot (k \cdot n)) + n$. $2 \cdot (k \cdot n)$ is even and n is odd. Thus $(2 \cdot (k \cdot n)) + n$ is odd. Therefore $n^{m'+1}$ is odd. Consequently $m' + 1 \in \Phi$. Qed.

Hence Φ contains every natural number (by `ARITHMETIC_01_4764664342773760`). Thus n^m is odd. $\square$