Division

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Definition 0.1. Let n be a natural number and m be a nonzero divisor of n. $\frac{n}{m}$ is the natural number k such that $k \cdot m = n$.

Let the quotient of n and m stand for $\frac{n}{m}$.

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Proposition 0.2. Let n be a natural number. $\frac{n}{1} = n$.

Proof. We have $\frac{n}{1} = \frac{n}{1} \cdot 1 = n$. □

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Proposition 0.3. Let n be a natural number and m be a nonzero divisor of n. Then $\frac{n}{m} = 0$ iff $n = 0$.

Proof. Case $\frac{n}{m} = 0$. Then $n = \frac{n}{m} \cdot m = 0 \cdot m = 0$. End.

Case $n = 0$. Then $\frac{n}{m} \cdot m = n = 0$. Hence $\frac{n}{m} = 0$ or $m = 0$. m is nonzero. Thus $\frac{n}{m} = 0$. End. □

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Proposition 0.4. Let n, m, k be natural numbers such that k is nonzero. Assume $k \mid n, m$. Then

$$\frac{n+m}{k} = \frac{n}{k} + \frac{m}{k}.$$

Proof. We have $\frac{n+m}{k} \cdot k = n + m$ and $\frac{n}{k} \cdot k = n$ and $\frac{m}{k} \cdot k = m$. Hence

$$\frac{n+m}{k} \cdot k = \left(\frac{n}{k} \cdot k\right) + \left(\frac{m}{k} \cdot k\right) = \left(\frac{n}{k} + \frac{m}{k}\right) \cdot k.$$

Thus $\frac{n+m}{k} = \frac{n}{k} + \frac{m}{k}$. □

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Proposition 0.5. Let n, m be natural numbers and k be a nonzero divisor of m. Then

$$\frac{n \cdot m}{k} = n \cdot \frac{m}{k}.$$

Proof. We have $\frac{n\cdot m}{k}\cdot k = n\cdot m$ and $\frac{m}{k}\cdot k = m$. Hence

$$\frac{n\cdot m}{k}\cdot k = n\cdot\left(\frac{m}{k}\cdot k\right) = \left(n\cdot\frac{m}{k}\right)\cdot k.$$

Thus $\frac{n\cdot m}{k} = n\cdot\frac{m}{k}$. □

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Corollary 0.6. Let n, m be natural numbers and k be a nonzero divisor of m. Then

$$\frac{n\cdot m}{k} = \frac{m}{k}\cdot n.$$

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Corollary 0.7. Let n, m be natural numbers and k be a nonzero divisor of n. Then

$$\frac{n\cdot m}{k} = \frac{n}{k}\cdot m.$$

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Corollary 0.8. Let n, m be natural numbers and k be a nonzero divisor of n. Then

$$\frac{n\cdot m}{k} = m\cdot\frac{n}{k}.$$

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Proposition 0.9. Let n, k be natural numbers such that k be nonzero and m be a nonzero divisor of n. Then

$$\frac{n\cdot k}{m\cdot k} = \frac{n}{m}.$$

Proof. We have $\frac{n\cdot k}{m\cdot k}\cdot(m\cdot k) = n\cdot k$. Hence

$$\left(\frac{n\cdot k}{m\cdot k}\cdot m\right)\cdot k = \frac{n\cdot k}{m\cdot k}\cdot(m\cdot k) = n\cdot k.$$

Thus $\frac{n\cdot k}{m\cdot k}\cdot m = n$. Therefore $\frac{n}{m} = \frac{n\cdot k}{m\cdot k}$. □

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Corollary 0.10. Let n, k be natural numbers such that k be nonzero and m be a nonzero divisor of n. Then

$$\frac{k \cdot n}{k \cdot m} = \frac{n}{m}.$$