

Russell's Paradox

Naproche formalization:

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Russell's Paradox is a set-theoretical paradox discovered by Bertrand Russell around 1902 [Frege1980] which shows that there exist statements φ whose extension $\{x \mid \varphi(x)\}$ cannot constitute a set – or in other words: Not every class is a set.

Theorem (Russell's Paradox). It is wrong that every class is a set.

Proof. Assume the contrary. Define $R = \{x \mid x \text{ is a set such that } x \notin x\}$.

Then R is a set. Hence $R \in R$ iff $R \notin R$. Contradiction. \square

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