Concatenation

LISTS_CONCAT_4578620297183232

Signature 0.1. Let L, L' be lists. $L +\!\!+ L'$ is a list.

LISTS_CONCAT_3703161885818880

Axiom 0.2. Let L be a list. Then $[] +\!\!+ L = L$.

LISTS_CONCAT_8050301789536256

Axiom 0.3. Let a be an object and L, L' be lists. Then $(a :: L) +\!\!+ L' = a :: (L +\!\!+ L')$.

LISTS_CONCAT_4512036658964875

Proposition 0.4. $L +\!\!+ [] = L$ for every list L.

Proof by induction on L. Let L be a list.

Case $L = []$. Trivial.

Case $L = a :: L'$ for some object a and some list L'. Consider an object a and a list L' such that $L = a :: L'$. Then $L' \prec L$. Hence $L' +\!\!+ [] = L'$. Thus $L +\!\!+ [] = (a :: L') +\!\!+ [] = a :: (L' +\!\!+ []) = a :: L' = L$. End. □

LISTS_CONCAT_1021563255448756

Proposition 0.5. $L +\!\!+ (L' +\!\!+ L'') = (L +\!\!+ L') +\!\!+ L''$ for any lists L, L', L''.

Proof by induction on L. Let L be a list.

Case $L = []$. Trivial.

Case $L = a :: L'''$ for some object a and some list L'''. Consider an object a and a list L''' such that $L = a :: L'''$. Then $L''' \prec L$. Let L', L'' be lists. Then $L''' +\!\!+ (L' +\!\!+ L'') = (L''' +\!\!+ L') +\!\!+ L''$. [prover vampire] Thus $L +\!\!+ (L' +\!\!+ L'') = (a :: L''') +\!\!+ (L' +\!\!+ L'') = a :: (L''' +\!\!+ (L' +\!\!+ L'')) = a :: ((L''' +\!\!+ L') +\!\!+ L'') = (a :: (L''' +\!\!+ L')) +\!\!+ L'' = ((a :: L''') +\!\!+ L') +\!\!+ L'' = (L +\!\!+ L') +\!\!+ L''$. End. □