Reversion

LISTS_REV_4578620297183232

Signature 0.1. Let L be a list. $\mathrm{rev}(L)$ is a list.

LISTS_REV_3703161885818880

Axiom 0.2. Let L be a list. Then $\mathrm{rev}([]) = []$.

LISTS_REV_8050301789536256

Axiom 0.3. Let a be an object and L be a list. Then $\mathrm{rev}(a :: L) = \mathrm{rev}(L) ++ [a]$.

LISTS_REV_4512036658964875

Proposition 0.4. $\mathrm{rev}(L ++ L') = \mathrm{rev}(L') ++ \mathrm{rev}(L)$ for any lists L, L'.

Proof by induction on L. Let L, L' be lists.

Case $L = []$. Trivial.

Case $L = a :: L''$ for some object a and some list L''. Take an object a and a list L'' such that $L = a :: L''$. Then $L'' \prec L$. Hence $\mathrm{rev}(L'' ++ L') = \mathrm{rev}(L') ++ \mathrm{rev}(L'')$. Thus $\mathrm{rev}(L ++ L') = \mathrm{rev}((a :: L'') ++ L') = \mathrm{rev}(a :: (L'' ++ L')) = \mathrm{rev}(L'' ++ L') ++ [a] = (\mathrm{rev}(L') ++ \mathrm{rev}(L'')) ++ [a] = \mathrm{rev}(L') ++ (\mathrm{rev}(L'') ++ [a]) = \mathrm{rev}(L') ++ \mathrm{rev}(a :: L'') = \mathrm{rev}(L') ++ \mathrm{rev}(L)$. Indeed $(\mathrm{rev}(L') ++ \mathrm{rev}(L'')) ++ [a] = \mathrm{rev}(L') ++ (\mathrm{rev}(L'') ++ [a])$ (by `LISTS_CONCAT_1021563255448756`). End. $\square$

LISTS_REV_1021563255448756

Proposition 0.5. $\mathrm{rev}(\mathrm{rev}(L)) = L$ for every list L.

Proof by induction on L. Let L be a list.

Case $L = []$. Trivial.

Case $L = a :: L'$ for some object a and some list L'. Take an object a and a list L' such that $L = a :: L'$. Then $L' \prec L$. Hence $\mathrm{rev}(\mathrm{rev}(L')) = L'$. Thus $\mathrm{rev}(\mathrm{rev}(L)) = \mathrm{rev}(\mathrm{rev}(a :: L')) = \mathrm{rev}(\mathrm{rev}(L') ++ [a]) = \mathrm{rev}([a]) ++ \mathrm{rev}(\mathrm{rev}(L')) = a :: \mathrm{rev}(\mathrm{rev}(L')) = a :: L' = L$. End. $\square$