Factorial

ARITHMETIC_12_0210357812531785

Signature 0.1. Let n be a natural number. $n!$ is a natural number.

ARITHMETIC_12_6945210541236989

Axiom 0.2. $0! = 1$.

ARITHMETIC_12_2312015846987452

Axiom 0.3. Let n be a natural number. Then $(n+1)! = n! \cdot (n+1)$.

ARITHMETIC_12_6123519005949952

Proposition 0.4. Let n be a natural number. Then $n! \neq 0$.

Proof. Define $\Phi = \{n' \in \mathbb{N} \mid n'! \neq 0\}$.

(1) Φ contains 0. Indeed $0! = 1 \neq 0$.

(2) For all $n' \in \Phi$ we have $n' + 1 \in \Phi$.
Proof. Let $n' \in \Phi$. We have $(n'+1)! = (n'+1) \cdot n'!$. $n'+1$ and $n'!$ are nonzero. Hence $(n'+1)! \neq 0$. Qed.

Thus Φ contains every natural number (by ARITHMETIC_01_4764664342773760). Therefore $n! \neq 0$. ∎