

# Russell's Paradox

Naproche formalization:

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2024

*Russell's Paradox* is a set-theoretical paradox discovered by Bertrand Russell around 1902 [Frege1980] which shows that there exist statements  $\varphi$  whose extension  $\{x \mid \varphi(x)\}$  cannot constitute a set – or in other words: Not every class is a set.

**Theorem (Russell's Paradox).** It is wrong that every class is a set.

*Proof.* Assume the contrary. Define  $R = \{x \mid x \text{ is a set such that } x \notin x\}$ .

Then  $R$  is a set. Hence  $R \in R$  iff  $R \notin R$ . Contradiction.  $\square$

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