Limits

Let $\lim x$ stand for $\bigcup x$.

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Proposition 0.1. Let x be a subset of **Ord**. Then $\lim x$ is an ordinal.

Proof. (1) $\lim x$ is transitive.
Proof. Let $y \in \lim x$ and $z \in y$. Take $w \in x$ such that $y \in w$. Hence w is transitive. Thus $z \in w$. Therefore $z \in \lim x$. Qed.

(2) Every element of $\lim x$ is transitive.
Proof. Let $y \in \lim x$. Let $z \in y$ and $v \in z$. Take $w \in x$ such that $y \in w$. Hence w is an ordinal. Thus y is an ordinal. Therefore y is transitive. Consequently $v \in y$. Qed. $\square$

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Definition 0.2. A limit ordinal is an ordinal λ such that neither λ is a successor ordinal nor $\lambda = 0$.