Countable and Uncountable Classes

Countably Infinite Classes

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Definition 0.1. Let X be a class. X is countably infinite iff X is equinumerous to $\mathbb{N}$.

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Proposition 0.2. Let X, Y be classes. If X is countably infinite and Y is equinumerous to X then Y is countably infinite.

Proof. Assume that X is countably infinite and Y is equinumerous to X. Take a bijection f between $\mathbb{N}$ and X and a bijection g between X and Y. Then $g \circ f$ is a bijection between $\mathbb{N}$ and Y (by `FOUNDATIONS_08_6435206693126144`). Indeed X, Y are classes. Hence Y is countably infinite. $\square$

Countable Classes

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Definition 0.3. Let X be a class. X is countable iff X is finite or X is countably infinite.

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Proposition 0.4. Let X, Y be classes. If X is countable and Y is equinumerous to X then Y is countable.

Proof. Assume that X is countable and Y is equinumerous to X. If X is finite then Y is finite. If X is countably infinite then Y is countably infinite. Hence Y is countable. $\square$

Uncountable Classes

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Definition 0.5. Let X be a class. X is uncountable iff X is not countable.

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Proposition 0.6. Let X, Y be classes. If X is uncountable and Y is equinumerous to X then Y is uncountable.

Proof. Assume that Y is equinumerous to X. If Y is countable then X is countable. Hence if X is uncountable then Y is uncountable. $\square$