

# Little Gauß' Theorem

Naproche formalization:

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This is a formalization of “Little Gauß' Theorem”, i.e. of the assertion that

$$\sum_{k=0}^n k = \frac{k(k+1)}{2}$$

for all  $n \in \mathbb{N}$ . In Naproche we can define the sum  $\sum_{n=0}^k n$  by the function  $\mathcal{G} : \mathbb{N} \rightarrow \mathbb{N}$  with  $\mathcal{G}(0) = 0$  and  $\mathcal{G}(n+1) = \mathcal{G}(n) + (n+1)$ :

**Signature.** Let  $n$  be a natural number.  $\mathcal{G}(n)$  is a natural number.

**Axiom.**  $\mathcal{G}(0) = 0$ .

**Axiom.** Let  $n$  be a natural number. Then  $\mathcal{G}(n+1) = \mathcal{G}(n) + (n+1)$ .

Then the theorem can be stated as follows.

**Theorem (Little Gauß).** For all natural numbers  $n$  we have

$$2 \cdot \mathcal{G}(n) = n \cdot (n+1).$$

*Proof.* Define  $\Phi = \{n \in \mathbb{N} \mid 2 \cdot \mathcal{G}(n) = n \cdot (n+1)\}$ .

(1)  $0 \in \Phi$ .

(2) For all  $n \in \Phi$  we have  $n+1 \in \Phi$ .

*Proof.* Let  $n \in \Phi$ . Then  $2 \cdot \mathcal{G}(n) = n \cdot (n+1)$ . Hence  $2 \cdot \mathcal{G}(n+1) = 2 \cdot (\mathcal{G}(n) + (n+1)) = (2 \cdot \mathcal{G}(n)) + (2 \cdot (n+1)) = (n \cdot (n+1)) + (2 \cdot (n+1)) = ((n+1) \cdot n) + ((n+1) \cdot 2) = (n+1) \cdot (n+2) = (n+1) \cdot ((n+1) + 1)$ . Therefore  $n+1 \in \Phi$ . Qed.

Thus  $\Phi$  contains every natural number. Consequently  $2 \cdot \mathcal{G}(n) = n \cdot (n+1)$  for every  $n \in \mathbb{N}$ .  $\square$

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