

The Russell-Myhill Paradox

Naproche formalization:

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The *Russell-Myhill Paradox* is a paradox discovered by Bertrand Russell which states that two systems of propositions with the same *logical product* are not necessarily equal. It was discussed in Russell's 1903 *Principles of Mathematics* [Russell1903] and rediscovered in 1958 by John Myhill [Myhill1958].

Signature. Let P be a system of propositions. The logical product of P is a proposition p such that p is true iff every element of P is true.

Let $\Box P$ denote the logical product of P .

Theorem (Russell-Myhill Paradox). It is wrong that

$$\Box P = \Box Q \implies P = Q$$

for all systems of propositions P, Q .

Proof. Assume the contrary. Define $Q = \{q \mid \text{there exists a system of propositions } P \text{ such that } q = \Box P \text{ and } q \notin P\}$. Consider $q = \Box Q$. Then for any system of propositions P such that $\Box q = P$ we have $P = Q$. Hence $q \in Q$ iff $q \notin Q$. Contradiction. \square

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