

1 Foundation

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Axiom 1.1 (Foundation). Let X be a nonempty system of sets. Then X has an element x such that X and x are disjoint.

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Corollary 1.2. Let X be a nonempty system of sets. Then X has an element x such that for no $y \in X$ we have $y \in x$.

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Proposition 1.3. Let Φ be a class. Assume that for all sets x if Φ contains every element of x that is a set then Φ contains x . Then Φ contains every set.

Proof. Assume the contrary. Define $M = \{x \mid x \text{ is a set such that } x \notin \Phi\}$. Then M is nonempty. Hence we can take a $x \in M$ such that for no $y \in M$ we have $y \in x$. Then x is a set such that every element of x that is a set is contained in Φ . Thus Φ contains x . Contradiction. \square