

# Java Source and Bytecode Formalizations in Isabelle: $\mu$ Java

Gerwin Klein    Tobias Nipkow    David von Oheimb    Cornelia Pusch  
Martin Strecker

January 18, 2026



# Contents

|          |  |           |
|----------|--|-----------|
| <b>1</b> | <b>Preface</b>   | <b>5</b>  |
| 1.1      | Introduction . . . . .   | 5         |
| 1.2      | Theory Dependencies . . . . .  | 7         |
| <b>2</b> | <b>Java Source Language</b>  | <b>9</b>  |
| 2.1      | Some Auxiliary Definitions . . . . .   | 9         |
| 2.2      | Java types . . . . .   | 10        |
| 2.3      | Class Declarations and Programs . . . . .  | 13        |
| 2.4      | Relations between Java Types . . . . .   | 14        |
| 2.5      | Java Values . . . . .  | 17        |
| 2.6      | Program State . . . . .  | 18        |
| 2.7      | Expressions and Statements . . . . .   | 20        |
| 2.8      | System Classes . . . . .   | 21        |
| 2.9      | Well-formedness of Java programs . . . . .   | 21        |
| 2.10     | Well-typedness Constraints . . . . .   | 27        |
| 2.11     | Operational Evaluation (big step) Semantics . . . . .  | 31        |
| 2.12     | Conformity Relations for Type Soundness Proof . . . . .  | 35        |
| 2.13     | Type Safety Proof . . . . .  | 39        |
| 2.14     | Example MicroJava Program . . . . .  | 41        |
| 2.15     | Example for generating executable code from Java semantics . . . . .                                 | 47        |
| <b>3</b> | <b>Java Virtual Machine</b>  | <b>51</b> |
| 3.1      | State of the JVM . . . . .   | 51        |
| 3.2      | Instructions of the JVM . . . . .  | 52        |
| 3.3      | JVM Instruction Semantics . . . . .  | 52        |
| 3.4      | Exception handling in the JVM . . . . .  | 55        |
| 3.5      | Program Execution in the JVM . . . . .   | 55        |
| 3.6      | Example for generating executable code from JVM semantics . . . . .                                  | 56        |
| 3.7      | A Defensive JVM . . . . .  | 59        |
| <b>4</b> | <b>Bytecode Verifier</b>   | <b>63</b> |
| 4.1      | Semilattices . . . . .   | 63        |
| 4.2      | The Error Type . . . . .   | 66        |
| 4.3      | Fixed Length Lists . . . . .   | 70        |
| 4.4      | Typing and Dataflow Analysis Framework . . . . .   | 76        |
| 4.5      | Products as Semilattices . . . . .   | 76        |
| 4.6      | More on Semilattices . . . . .   | 78        |
| 4.7      | Lifting the Typing Framework to <code>err</code> , <code>app</code> , and <code>eff</code> . . . . . | 79        |

|      |  |     |
|------|--|-----|
| 4.8  | Kildall's Algorithm  | 81  |
| 4.9  | More about Options   | 84  |
| 4.10 | The Lightweight Bytecode Verifier  | 86  |
| 4.11 | Correctness of the LBV   | 90  |
| 4.12 | Completeness of the LBV  | 92  |
| 4.13 | Abstract Bytecode Verifier   | 95  |
| 4.14 | Semilattices   | 95  |
| 4.15 | The Java Type System as Semilattice  | 95  |
| 4.16 | The JVM Type System as Semilattice   | 96  |
| 4.17 | Effect of Instructions on the State Type   | 101 |
| 4.18 | Monotonicity of eff and app  | 107 |
| 4.19 | The Bytecode Verifier  | 108 |
| 4.20 | The Typing Framework for the JVM   | 110 |
| 4.21 | LBV for the JVM  | 113 |
| 4.22 | BV Type Safety Invariant   | 115 |
| 4.23 | BV Type Safety Proof   | 119 |
| 4.24 | Welltyped Programs produce no Type Errors  | 126 |
| 4.25 | Kildall for the JVM  | 130 |
| 4.26 | Example Welltypings  | 131 |
| 4.27 | Alternative definition of well-typing of bytecode, used in compiler type correctness proof | 162 |

# Chapter 1

## Preface

### 1.1 Introduction

This document contains the automatically generated listings of the Isabelle sources for  $\mu$ Java.  $\mu$ Java is a reduced model of JavaCard, dedicated to the study of the interaction of the source language, byte code, the byte code verifier and the compiler. In order to make the Isabelle sources more accessible, this introduction provides a brief survey of the main concepts of  $\mu$ Java.

The  $\mu$ Java **source language** (see Chapter 2) only comprises a part of the original JavaCard language. It models features such as:

- The basic “primitive types” of Java
- Object orientation, in particular classes, and relevant relations on classes (subclass, widening)
- Methods and method signatures
- Inheritance and overriding of methods, dynamic binding
- Representatives of “relevant” expressions and statements
- Generation and propagation of system exceptions

However, the following features are missing in  $\mu$ Java wrt. JavaCard:

- Some primitive types (**byte**, **short**)
- Interfaces and related concepts, arrays
- Most numeric operations, syntactic variants of statements (**do-loop**, **for-loop**)
- Complex block structure, method bodies with multiple returns
- Abrupt termination (**break**, **continue**)
- Class and method modifiers (such as **static** and **public/private** access modifiers)
- User-defined exception classes and an explicit **throw**-statement. Exceptions cannot be caught.

- A “definite assignment” check

In addition, features are missing that are not part of the JavaCard language, such as multithreading and garbage collection. No attempt has been made to model peculiarities of JavaCard such as the applet firewall or the transaction mechanism.

For a more complete Isabelle model of JavaCard, the reader should consult the Bali formalization (<https://isabelle.in.tum.de/verificard/Bali/document.pdf>), which models most of the source language features of JavaCard, however without describing the bytecode level.

The central topics of the source language formalization are:

- Description of the structure of the “runtime environment”, in particular structure of classes and the program state
- Definition of syntax, typing rules and operational semantics of statements and expressions
- Definition of “conformity” (characterizing type safety) and a type safety proof

The  $\mu$ Java **virtual machine** (see Chapter 3) corresponds rather directly to the source level, in the sense that the same data types are supported and bytecode instructions required for emulating the source level operations are provided. Again, only one representative of different variants of instructions has been selected; for example, there is only one comparison operator. The formalization of the bytecode level is purely descriptive (“no theorems”) and rather brief as compared to the source level; all questions related to type systems for and type correctness of bytecode are dealt with in chapter on bytecode verification.

The problem of **bytecode verification** (see Chapter 4) is dealt with in several stages:

- First, the notion of “method type” is introduced, which corresponds to the notion of “type” on the source level.
- Well-typedness of instructions wrt. a method type is defined (see Section 4.19). Roughly speaking, determining well-typedness is *type checking*.
- It is shown that bytecode that is well-typed in this sense can be safely executed – a type soundness proof on the bytecode level (Section 4.23).
- Given raw bytecode, one of the purposes of bytecode verification is to determine a method type that is well-typed according to the above definition. Roughly speaking, this is *type inference*. The Isabelle formalization presents bytecode verification as an instance of an abstract dataflow algorithm (Kildall’s algorithm, see Sections 4.8 to 4.25).

Bytecode verification in  $\mu$ Java so far takes into account:

- Operations and branching instructions
- Exceptions

Initialization during object creation is not accounted for in the present document (see the formalization in <https://isabelle.in.tum.de/verificard/obj-init/document.pdf>), neither is the `jsr` instruction.

## 1.2 Theory Dependencies

Figure [1.1](#) shows the dependencies between the Isabelle theories in the following sections.

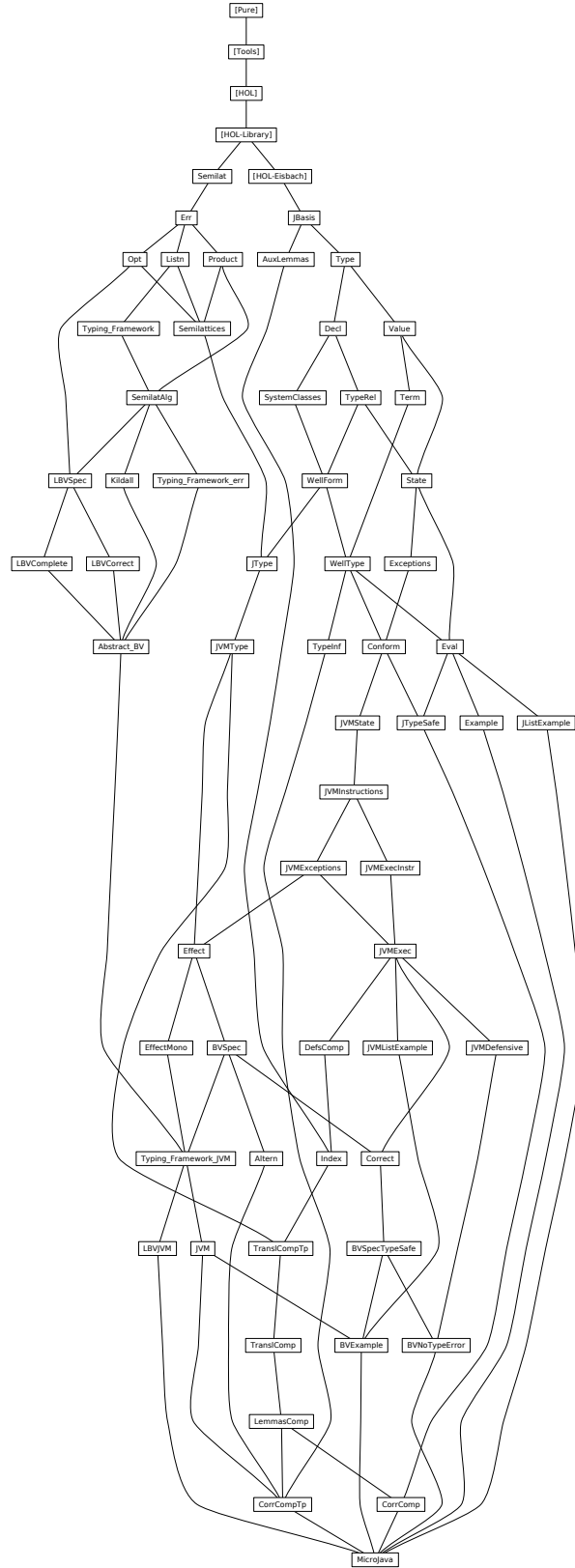


Figure 1.1: Theory Dependency Graph



## Chapter 2

# Java Source Language

### 2.1 Some Auxiliary Definitions

```
theory JBasis
imports
  Main
  "HOL-Library.Transitive_Closure_Table"
  "HOL-Eisbach.Eisbach"
begin

lemmas [simp] = Let_def
```

#### 2.1.1 unique

```
definition unique :: "('a × 'b) list => bool" where
  "unique == distinct ∘ map fst"
```

```
lemma fst_in_set_lemma: "(x, y) ∈ set xys ⟹ x ∈ fst ` set xys"
  ⟨proof⟩
```

```
lemma unique_Nil [simp]: "unique []"
  ⟨proof⟩
```

```
lemma unique_Cons [simp]: "unique ((x,y)#l) = (unique l & (∀y. (x,y) ∉ set l))"
  ⟨proof⟩
```

```
lemma unique_append: "unique l' ⟹ unique l ⟹
  (∀ (x,y) ∈ set l. ∀ (x',y') ∈ set l'. x' ≠ x) ⟹ unique (l @ l')"
  ⟨proof⟩
```

```
lemma unique_map_inj: "unique l ⟹ inj f ⟹ unique (map (%(k,x). (f k, g k x)) l)"
  ⟨proof⟩
```

#### 2.1.2 More about Maps

```
lemma map_of_SomeI: "unique l ⟹ (k, x) ∈ set l ⟹ map_of l k = Some x"
  ⟨proof⟩
```

```
lemma Ball_set_table: "(∀ (x,y) ∈ set l. P x y) ⟹ (∀ x. ∀ y. map_of l x = Some y ⟶ P
```

10

```
x y)"  
  ⟨proof⟩
```

```
lemma table_of_remap_SomeD:  
  "map_of (map (λ((k,k'),x). (k,(k',x))) t) k = Some (k',x) ==>  
    map_of t (k, k') = Some x"  
  ⟨proof⟩  
  
end
```

## 2.2 Java types

```
theory Type imports JBasis begin
```

```
typeddecl cnam
```

```
instantiation cnam :: equal  
begin
```

```
definition "HOL.equal (cn :: cnam) cn' ⟷ cn = cn'"  
instance ⟨proof⟩
```

```
end
```

These instantiations only ensure that the merge in theory *MicroJava* succeeds. FIXME

```
instantiation cnam :: typerep  
begin
```

```
definition "typerep_class.typerep ≡ λ_ :: cnam itself. Typerep.Typerep (STR ''Type.cnam'')  
  []"  
instance ⟨proof⟩
```

```
end
```

```
instantiation cnam :: term_of  
begin
```

```
definition "term_of_class.term_of (C :: cnam) ≡  
  Code_Evaluation.Const (STR ''dummy_cnam'') (Typerep.Typerep (STR ''Type.cnam'') [])"  
instance ⟨proof⟩
```

```
end
```

```
instantiation cnam :: partial_term_of  
begin
```

```
definition "partial_term_of_class.partial_term_of (C :: cnam itself) n = undefined"  
instance ⟨proof⟩
```

```
end
```

```
— exceptions  
datatype
```

```

    xcpt
    = NullPointer
    | ClassCast
    | OutOfMemory

— class names
datatype cname
  = Object
  | Xcpt xcpt
  | Cname cnam

typedecl vnam — variable or field name

instantiation vnam :: equal
begin

definition "HOL.equal (vn :: vnam) vn'  $\longleftrightarrow$  vn = vn'"
instance <proof>

end

instantiation vnam :: typerep
begin

definition "typerep_class.typerep  $\equiv$   $\lambda\_ :: vnam$  itself. Typerep.Typerep (STR ''Type.vnam'')
  []"
instance <proof>

end

instantiation vnam :: term_of
begin

definition "term_of_class.term_of (V :: vnam)  $\equiv$ 
  Code_Evaluation.Const (STR ''dummy_vnam'') (Typerep.Typerep (STR ''Type.vnam'') [])"
instance <proof>

end

instantiation vnam :: partial_term_of
begin

definition "partial_term_of_class.partial_term_of (V :: vnam itself) n = undefined"
instance <proof>

end

typedecl mname — method name

instantiation mname :: equal
begin

definition "HOL.equal (M :: mname) M'  $\longleftrightarrow$  M = M'"
instance <proof>

```

end

instantiation *mname* :: *typerep*  
begin

definition "*typerep\_class.typerep*  $\equiv \lambda\_ :: mname\ itself.\ Typerep.Typerep\ (STR\ \text{'Type.mname'})$   
[]"

instance  $\langle proof \rangle$

end

instantiation *mname* :: *term\_of*  
begin

definition "*term\_of\_class.term\_of* (*M* :: *mname*)  $\equiv$   
Code\_Evaluation.Const (STR 'dummy\_mname') (Typerep.Typerep (STR 'Type.mname') [])"

instance  $\langle proof \rangle$

end

instantiation *mname* :: *partial\_term\_of*  
begin

definition "*partial\_term\_of\_class.partial\_term\_of* (*M* :: *mname* itself) *n* = undefined"

instance  $\langle proof \rangle$

end

— names for *This* pointer and local/field variables

**datatype** *vname*  
= *This*  
/ *VName vnam*

— primitive type, cf. 4.2

**datatype** *prim\_ty*  
= *Void* — 'result type' of void methods  
/ *Boolean*  
/ *Integer*

— reference type, cf. 4.3

**datatype** *ref\_ty*  
= *NullT* — null type, cf. 4.1  
/ *ClassT cname* — class type

— any type, cf. 4.1

**datatype** *ty*  
= *PrimT prim\_ty* — primitive type  
/ *RefT ref\_ty* — reference type

**abbreviation** *NT* :: *ty*  
where "*NT* == *RefT NullT*"

**abbreviation** *Class* :: "*cname* => *ty*"

```

    where "Class C == RefT (ClassT C)"

end

```

## 2.3 Class Declarations and Programs

```
theory Decl imports Type begin
```

```
type_synonym
  fdecl      = "vname × ty"          — field declaration, cf. 8.3 (, 9.3)
```

```
type_synonym
  sig         = "mname × ty list"    — signature of a method, cf. 8.4.2
```

```
type_synonym
  'c mdecl    = "sig × ty × 'c"      — method declaration in a class
```

```
type_synonym
  'c "class"  = "cname × fdecl list × 'c mdecl list"
  — class = superclass, fields, methods
```

```
type_synonym
  'c cdecl    = "cname × 'c class"   — class declaration, cf. 8.1
```

```
type_synonym
  'c prog     = "'c cdecl list"     — program
```

```
translations
  (type) "fdecl" <= (type) "vname × ty"
  (type) "sig" <= (type) "mname × ty list"
  (type) "'c mdecl" <= (type) "sig × ty × 'c"
  (type) "'c class" <= (type) "cname × fdecl list × ('c mdecl) list"
  (type) "'c cdecl" <= (type) "cname × ('c class)"
  (type) "'c prog" <= (type) "('c cdecl) list"
```

```
definition "class" :: "'c prog => (cname → 'c class)" where
  "class ≡ map_of"
```

```
definition is_class :: "'c prog => cname => bool" where
  "is_class G C ≡ class G C ≠ None"
```

```
lemma finite_is_class: "finite {C. is_class G C}"
⟨proof⟩
```

```
primrec is_type :: "'c prog => ty => bool" where
  "is_type G (PrimT pt) = True"
| "is_type G (RefT t) = (case t of NullT => True | ClassT C => is_class G C)"
```

```
end
```

## 2.4 Relations between Java Types

**theory** TypeRel

**imports** Decl

**begin**

— direct subclass, cf. 8.1.3

**inductive\_set**

subcls1 :: "'c prog => (cname × cname) set"

and subcls1' :: "'c prog => cname => cname => bool" ( $\langle \_ \vdash \_ \prec_{C1} \_ \rangle$  [71,71,71] 70)

for G :: "'c prog"

**where**

"G ⊢ C <C1 D ≡ (C, D) ∈ subcls1 G"

| subcls1I: "⌊class G C = Some (D,rest); C ≠ Object⌋ ⇒ G ⊢ C <C1 D"

**abbreviation**

subcls :: "'c prog => cname => cname => bool" ( $\langle \_ \vdash \_ \preceq_C \_ \rangle$  [71,71,71] 70)

**where** "G ⊢ C ⪯C D ≡ (C, D) ∈ (subcls1 G)\*"

**lemma** subcls1D:

"G ⊢ C <C1 D ⇒ C ≠ Object ∧ (∃ fs ms. class G C = Some (D,fs,ms))"

⟨proof⟩

**lemma** subcls1\_def2:

"subcls1 P =

(SIGMA C:{C. is\_class P C}. {D. C ≠ Object ∧ fst (the (class P C))=D})"

⟨proof⟩

**lemma** finite\_subcls1: "finite (subcls1 G)"

⟨proof⟩

**lemma** subcls\_is\_class: "(C, D) ∈ (subcls1 G)<sup>+</sup> ⇒ is\_class G C"

⟨proof⟩

**lemma** subcls\_is\_class2 [rule\_format (no\_asm)]:

"G ⊢ C ⪯C D ⇒ is\_class G D → is\_class G C"

⟨proof⟩

**definition** class\_rec :: "'c prog => cname => 'a =>

(cname => fdecl list => 'c mdecl list => 'a => 'a) => 'a" **where**

"class\_rec G == wfrec ((subcls1 G)<sup>-1</sup>)

(λr C t f. case class G C of

None => undefined

| Some (D,fs,ms) =>

f C fs ms (if C = Object then t else r D t f))"

**lemma** class\_rec\_lemma:

**assumes** wf: "wf ((subcls1 G)<sup>-1</sup>)"

**and** cls: "class G C = Some (D, fs, ms)"

**shows** "class\_rec G C t f = f C fs ms (if C=Object then t else class\_rec G D t f)"

⟨proof⟩

**definition**

```
"wf_class G = wf ((subcls1 G)-1)"
```

Code generator setup

```
code_pred
  (modes: i ⇒ i ⇒ o ⇒ bool, i ⇒ i ⇒ i ⇒ bool)
  subcls1p
  ⟨proof⟩
```

```
declare subcls1_def [code_pred_def]
```

```
code_pred
  (modes: i ⇒ i × o ⇒ bool, i ⇒ i × i ⇒ bool)
  [inductify]
  subcls1
  ⟨proof⟩
```

```
definition subcls' where "subcls' G = (subcls1p G)**"
```

```
code_pred
  (modes: i ⇒ i ⇒ i ⇒ bool, i ⇒ i ⇒ o ⇒ bool)
  [inductify]
  subcls'
  ⟨proof⟩
```

```
lemma subcls_conv_subcls' [code_unfold]:
  "(subcls1 G)* = {(C, D). subcls' G C D}"
  ⟨proof⟩
```

```
lemma class_rec_code [code]:
  "class_rec G C t f =
    (if wf_class G then
      (case class G C of
        None ⇒ class_rec G C t f
      | Some (D, fs, ms) ⇒
        if C = Object then f Object fs ms t else f C fs ms (class_rec G D t f))
    else class_rec G C t f)"
  ⟨proof⟩
```

```
lemma wf_class_code [code]:
  "wf_class G ⟷ (∀ (C, rest) ∈ set G. C ≠ Object ⟶ ¬ G ⊢ fst (the (class G C)) ≤C C)"
  ⟨proof⟩
```

```
definition "method" :: "'c prog × cname => (sig → cname × ty × 'c)"
  — methods of a class, with inheritance, overriding and hiding, cf. 8.4.6
  where [code]: "method ≡ λ(G,C). class_rec G C Map.empty (λC fs ms ts.
    ts ++ map_of (map (λ(s,m). (s, (C,m))) ms))"
```

```
definition fields :: "'c prog × cname => ((vname × cname) × ty) list"
  — list of fields of a class, including inherited and hidden ones
  where [code]: "fields ≡ λ(G,C). class_rec G C [] (λC fs ms ts.
    map (λ(fn,ft). ((fn,C),ft)) fs @ ts)"
```

```
definition field :: "'c prog × cname => (vname → cname × ty)"
```

```

where [code]: "field == map_of o (map (λ((fn,fd),ft). (fn,(fd,ft)))) o fields"

lemma method_rec_lemma: "[|class G C = Some (D,fs,ms); wf ((subcls1 G)-1)|] ==>
  method (G,C) = (if C = Object then Map.empty else method (G,D)) ++
  map_of (map (λ(s,m). (s,(C,m))) ms)"
⟨proof⟩

lemma fields_rec_lemma: "[|class G C = Some (D,fs,ms); wf ((subcls1 G)-1)|] ==>
  fields (G,C) =
  map (λ(fn,ft). ((fn,C),ft)) fs @ (if C = Object then [] else fields (G,D))"
⟨proof⟩

lemma field_fields:
"field (G,C) fn = Some (fd, fT) ==> map_of (fields (G,C)) (fn, fd) = Some fT"
⟨proof⟩
inductive
  widen    :: "'c prog => [ty    , ty    ] => bool" (<_ ⊢ _ ≤ _> [71,71,71] 70)
  for G :: "'c prog"
where
  refl [intro!, simp]:      "G ⊢      T ≤ T"      — identity conv., cf. 5.1.1
| subcls      : "G ⊢ C ≤ C D ==> G ⊢ Class C ≤ Class D"
| null [intro!]:      "G ⊢      NT ≤ RefT R"

code_pred widen ⟨proof⟩

lemmas refl = HOL.refl

— casting conversion, cf. 5.5 / 5.1.5
— left out casts on primitive types
inductive
  cast      :: "'c prog => [ty    , ty    ] => bool" (<_ ⊢ _ ≤? _> [71,71,71] 70)
  for G :: "'c prog"
where
  widen:      "G ⊢ C ≤ D ==> G ⊢ C ≤? D"
| subcls:      "G ⊢ D ≤ C C ==> G ⊢ Class C ≤? Class D"

lemma widen_PrimT_RefT [iff]: "(G ⊢ PrimT pT ≤ RefT rT) = False"
⟨proof⟩

lemma widen_RefT: "G ⊢ RefT R ≤ T ==> ∃ t. T = RefT t"
⟨proof⟩

lemma widen_RefT2: "G ⊢ S ≤ RefT R ==> ∃ t. S = RefT t"
⟨proof⟩

lemma widen_Class: "G ⊢ Class C ≤ T ==> ∃ D. T = Class D"
⟨proof⟩

lemma widen_Class_NullT [iff]: "(G ⊢ Class C ≤ NT) = False"
⟨proof⟩

lemma widen_Class_Class [iff]: "(G ⊢ Class C ≤ Class D) = (G ⊢ C ≤ C D)"
⟨proof⟩

```



```
lemma widen_NT_Class [simp]: "G ⊢ T ≤ NT ⇒ G ⊢ T ≤ Class D"
⟨proof⟩
```

```
lemma cast_PrimT_RefT [iff]: "(G ⊢ PrimT pT ≤? RefT rT) = False"
⟨proof⟩
```

```
lemma cast_RefT: "G ⊢ C ≤? Class D ⇒ ∃ rT. C = RefT rT"
⟨proof⟩
```

```
theorem widen_trans[trans]: "[G ⊢ S ≤ U; G ⊢ U ≤ T] ⇒ G ⊢ S ≤ T"
⟨proof⟩
```

```
end
```

## 2.5 Java Values

```
theory Value imports Type begin
```

```
typeddecl loc' — locations, i.e. abstract references on objects
```

```
datatype loc
  = XcptRef xcpt — special locations for pre-allocated system exceptions
  | Loc loc' — usual locations (references on objects)
```

```
datatype val
  = Unit — dummy result value of void methods
  | Null — null reference
  | Bool bool — Boolean value
  | Intg int — integer value, name Intg instead of Int because of clash with HOL/Set.thy
  | Addr loc — addresses, i.e. locations of objects
```

```
primrec the_Bool :: "val ⇒ bool" where
  "the_Bool (Bool b) = b"
```

```
primrec the_Intg :: "val ⇒ int" where
  "the_Intg (Intg i) = i"
```

```
primrec the_Addr :: "val ⇒ loc" where
  "the_Addr (Addr a) = a"
```

```
primrec defpval :: "prim_ty ⇒ val" — default value for primitive types where
  "defpval Void = Unit"
| "defpval Boolean = Bool False"
| "defpval Integer = Intg 0"
```

```
primrec default_val :: "ty ⇒ val" — default value for all types where
  "default_val (PrimT pt) = defpval pt"
| "default_val (RefT r) = Null"
```

```
end
```

## 2.6 Program State

```

theory State
imports TypeRel Value
begin

type_synonym
  fields' = "(vname × cname → val)" — field name, defining class, value

type_synonym
  obj = "cname × fields'" — class instance with class name and fields

definition obj_ty :: "obj ⇒ ty" where
  "obj_ty obj == Class (fst obj)"

definition init_vars :: "('a × ty) list ⇒ ('a → val)" where
  "init_vars == map_of o map (λ(n,T). (n,default_val T))"

type_synonym aheap = "loc → obj" — "heap" used in a translation below
type_synonym locals = "vname → val" — simple state, i.e. variable contents

type_synonym state = "aheap × locals" — heap, local parameter including This
type_synonym xstate = "val option × state" — state including exception information

abbreviation (input)
  heap :: "state ⇒ aheap"
  where "heap == fst"

abbreviation (input)
  locals :: "state ⇒ locals"
  where "locals == snd"

abbreviation "Norm s == (None, s)"

abbreviation (input)
  abrupt :: "xstate ⇒ val option"
  where "abrupt == fst"

abbreviation (input)
  store :: "xstate ⇒ state"
  where "store == snd"

abbreviation
  lookup_obj :: "state ⇒ val ⇒ obj"
  where "lookup_obj s a' == the (heap s (the_Addr a'))"

definition raise_if :: "bool ⇒ xcpt ⇒ val option ⇒ val option" where
  "raise_if b x xo ≡ if b ∧ (xo = None) then Some (Addr (XcptRef x)) else xo"

Make new_Addr completely specified (at least for the code generator)

consts nat_to_loc' :: "nat ⇒ loc'"
code_datatype nat_to_loc'
definition new_Addr :: "aheap ⇒ loc × val option" where
  "new_Addr h ≡

```

```

if  $\exists n. h \text{ (Loc (nat\_to\_loc' n)) = None}$ 
then (Loc (nat\_to\_loc' (LEAST n. h (Loc (nat\_to\_loc' n)) = None)), None)
else (Loc (nat\_to\_loc' 0), Some (Addr (XcptRef OutOfMemory)))"

```

**definition** np :: "val => val option => val option" where  
 "np v == raise\_if (v = Null) NullPointer"

**definition** c\_hupd :: "aheap => xstate => xstate" where  
 "c\_hupd h' ==  $\lambda(xo, (h, l)). \text{ if } xo = \text{None then } (\text{None}, (h', l)) \text{ else } (xo, (h, l))$ "

**definition** cast\_ok :: "'c prog => cname => aheap => val => bool" where  
 "cast\_ok G C h v ==  $v = \text{Null} \vee G \vdash \text{obj\_ty (the (h (the\_Addr v)))} \preceq \text{Class C}$ "

**lemma** obj\_ty\_def2 [simp]: "obj\_ty (C, fs) = Class C"  
 <proof>

**lemma** new\_AddrD: "new\_Addr hp = (ref, xcp)  $\implies$   
 hp ref = None  $\wedge$  xcp = None  $\vee$  xcp = Some (Addr (XcptRef OutOfMemory))"  
 <proof>

**lemma** raise\_if\_True [simp]: "raise\_if True x y  $\neq$  None"  
 <proof>

**lemma** raise\_if\_False [simp]: "raise\_if False x y = y"  
 <proof>

**lemma** raise\_if\_Some [simp]: "raise\_if c x (Some y)  $\neq$  None"  
 <proof>

**lemma** raise\_if\_Some2 [simp]:  
 "raise\_if c z (if x = None then Some y else x)  $\neq$  None"  
 <proof>

**lemma** raise\_if\_SomeD [rule\_format (no\_asm)]:  
 "raise\_if c x y = Some z  $\longrightarrow c \wedge \text{Some } z = \text{Some (Addr (XcptRef x))} \mid y = \text{Some } z$ "  
 <proof>

**lemma** raise\_if\_NoneD [rule\_format (no\_asm)]:  
 "raise\_if c x y = None  $\longrightarrow \neg c \wedge y = \text{None}$ "  
 <proof>

**lemma** np\_NoneD [rule\_format (no\_asm)]:  
 "np a' x' = None  $\longrightarrow x' = \text{None} \wedge a' \neq \text{Null}$ "  
 <proof>

**lemma** np\_None [rule\_format (no\_asm), simp]: "a'  $\neq$  Null  $\longrightarrow$  np a' x' = x'"  
 <proof>

**lemma** np\_Some [simp]: "np a' (Some xc) = Some xc"  
 <proof>

**lemma** np\_Null [simp]: "np Null None = Some (Addr (XcptRef NullPointer))"  
 <proof>

```
lemma np_Addr [simp]: "np (Addr a) None = None"
<proof>
```

```
lemma np_raise_if [simp]: "(np Null (raise_if c xc None)) =
  Some (Addr (XcptRef (if c then xc else NullPointer)))"
<proof>
```

```
lemma c_hupd_fst [simp]: "fst (c_hupd h (x, s)) = x"
<proof>
```

Naive implementation for `new_Addr` by exhaustive search

```
definition gen_new_Addr :: "aheap => nat => loc × val option" where
  "gen_new_Addr h n ≡
    if ∃ a. a ≥ n ∧ h (Loc (nat_to_loc' a)) = None
    then (Loc (nat_to_loc' (LEAST a. a ≥ n ∧ h (Loc (nat_to_loc' a)) = None)), None)
    else (Loc (nat_to_loc' 0), Some (Addr (XcptRef OutOfMemory)))"
```

```
lemma new_Addr_code_code [code]:
  "new_Addr h = gen_new_Addr h 0"
<proof>
```

```
lemma gen_new_Addr_code [code]:
  "gen_new_Addr h n = (if h (Loc (nat_to_loc' n)) = None then (Loc (nat_to_loc' n), None)
  else gen_new_Addr h (Suc n))"
<proof>
```

```
instantiation loc' :: equal
begin
```

```
definition "HOL.equal (l :: loc') l' ⟷ l = l'"
instance <proof>
```

```
end
```

```
end
```

## 2.7 Expressions and Statements

```
theory Term imports Value begin
```

```
datatype binop = Eq | Add      — function codes for binary operation
```

```
datatype expr
  = NewC cname                — class instance creation
  | Cast cname expr           — type cast
  | Lit val                   — literal value, also references
  | BinOp binop expr expr    — binary operation
  | LAcc vname                — local (incl. parameter) access
  | LAss vname expr           (<_ := _> [90,90]90) — local assign
  | FAcc cname expr vname    (<{ }_ . _> [10,90,99]90) — field access
  | FAss cname expr vname   (<{ }_ . _ := _> [10,90,99,90]90) — field ass.
  expr
```

```

| Call cname expr mname
  "ty list" "expr list"    (<{ }_...'_ { }_')> [10,90,99,10,10] 90) — method call

datatype_compat expr

datatype stmt
= Skip                      — empty statement
| Expr expr                 — expression statement
| Comp stmt stmt            (<_;; _> [61,60]60)
| Cond expr stmt stmt      (<If '(_)' _ Else _> [80,79,79]70)
| Loop expr stmt           (<While '(_)' _> [80,79]70)

end

```

## 2.8 System Classes

theory SystemClasses imports Decl begin

This theory provides definitions for the *Object* class, and the system exceptions.

```

definition ObjectC :: "'c cdecl" where
  [code_unfold]: "ObjectC ≡ (Object, (undefined, [], []))"

```

```

definition NullPointerC :: "'c cdecl" where
  [code_unfold]: "NullPointerC ≡ (Xcpt NullPointer, (Object, [], []))"

```

```

definition ClassCastC :: "'c cdecl" where
  [code_unfold]: "ClassCastC ≡ (Xcpt ClassCast, (Object, [], []))"

```

```

definition OutOfMemoryC :: "'c cdecl" where
  [code_unfold]: "OutOfMemoryC ≡ (Xcpt OutOfMemory, (Object, [], []))"

```

```

definition SystemClasses :: "'c cdecl list" where
  [code_unfold]: "SystemClasses ≡ [ObjectC, NullPointerC, ClassCastC, OutOfMemoryC]"

```

end

## 2.9 Well-formedness of Java programs

```

theory WellForm
imports TypeRel SystemClasses
begin

```

for static checks on expressions and statements, see WellType.

**improvements over Java Specification 1.0 (cf. 8.4.6.3, 8.4.6.4, 9.4.1):**

- a method implementing or overwriting another method may have a result type that widens to the result type of the other method (instead of identical type)

**simplifications:**

- for uniformity, Object is assumed to be declared like any other class

```
type_synonym 'c wf_mb = "'c prog => cname => 'c mdecl => bool"
```

```
definition wf_syscls :: "'c prog => bool" where
"wf_syscls G == let cs = set G in Object ∈ fst ' cs ∧ (∀x. Xcpt x ∈ fst ' cs)"
```

```
definition wf_fdecl :: "'c prog => fdecl => bool" where
"wf_fdecl G == λ(fn,ft). is_type G ft"
```

```
definition wf_mhead :: "'c prog => sig => ty => bool" where
"wf_mhead G == λ(mn,pTs) rT. (∀T∈set pTs. is_type G T) ∧ is_type G rT"
```

```
definition ws_cdecl :: "'c prog => 'c cdecl => bool" where
"ws_cdecl G ==
  λ(C,(D,fs,ms)).
  (∀f∈set fs. wf_fdecl G f) ∧ unique fs ∧
  (∀(sig,rT,mb)∈set ms. wf_mhead G sig rT) ∧ unique ms ∧
  (C ≠ Object → is_class G D ∧ ¬G⊢D⊆C C)"
```

```
definition ws_prog :: "'c prog => bool" where
"ws_prog G ==
  wf_syscls G ∧ (∀c∈set G. ws_cdecl G c) ∧ unique G"
```

```
definition wf_mrT :: "'c prog => 'c cdecl => bool" where
"wf_mrT G ==
  λ(C,(D,fs,ms)).
  (C ≠ Object → (∀(sig,rT,b)∈set ms. ∀D' rT' b'.
    method(G,D) sig = Some(D',rT',b') → G⊢rT⊆rT'))"
```

```
definition wf_cdecl_mdecl :: "'c wf_mb => 'c prog => 'c cdecl => bool" where
"wf_cdecl_mdecl wf_mb G ==
  λ(C,(D,fs,ms)). (∀m∈set ms. wf_mb G C m)"
```

```
definition wf_prog :: "'c wf_mb => 'c prog => bool" where
"wf_prog wf_mb G ==
  ws_prog G ∧ (∀c∈set G. wf_mrT G c ∧ wf_cdecl_mdecl wf_mb G c)"
```

```
definition wf_mdecl :: "'c wf_mb => 'c wf_mb" where
"wf_mdecl wf_mb G C == λ(sig,rT,mb). wf_mhead G sig rT ∧ wf_mb G C (sig,rT,mb)"
```

```
definition wf_cdecl :: "'c wf_mb => 'c prog => 'c cdecl => bool" where
"wf_cdecl wf_mb G ==
  λ(C,(D,fs,ms)).
  (∀f∈set fs. wf_fdecl G f) ∧ unique fs ∧
  (∀m∈set ms. wf_mdecl wf_mb G C m) ∧ unique ms ∧
  (C ≠ Object → is_class G D ∧ ¬G⊢D⊆C C ∧
    (∀(sig,rT,b)∈set ms. ∀D' rT' b'.
      method(G,D) sig = Some(D',rT',b') → G⊢rT⊆rT'))"
```

```
lemma wf_cdecl_mrT_cdecl_mdecl:
```

```
"(wf_cdecl wf_mb G c) = (ws_cdecl G c ∧ wf_mrT G c ∧ wf_cdecl_mdecl wf_mb G c)"
⟨proof⟩
```

**lemma** `wf_cdecl_ws_cdecl [intro]: "wf_cdecl wf_mb G cd  $\implies$  ws_cdecl G cd"`  
`<proof>`

**lemma** `wf_prog_ws_prog [intro]: "wf_prog wf_mb G  $\implies$  ws_prog G"`  
`<proof>`

**lemma** `wf_prog_wf_mdecl:`  
`"[[ wf_prog wf_mb G; (C,S,fs,mdecls)  $\in$  set G; ((mn,pTs),rT,code)  $\in$  set mdecls]]`  
 `$\implies$  wf_mdecl wf_mb G C ((mn,pTs),rT,code)"`  
`<proof>`

**lemma** `class_wf:`  
`"[|class G C = Some c; wf_prog wf_mb G|]`  
 `$\implies$  wf_cdecl wf_mb G (C,c)  $\wedge$  wf_mrT G (C,c)"`  
`<proof>`

**lemma** `class_wf_struct:`  
`"[|class G C = Some c; ws_prog G|]`  
 `$\implies$  ws_cdecl G (C,c)"`  
`<proof>`

**lemma** `class_Object [simp]:`  
`"ws_prog G  $\implies$   $\exists X$  fs ms. class G Object = Some (X,fs,ms)"`  
`<proof>`

**lemma** `class_Object_syscls [simp]:`  
`"wf_syscls G  $\implies$  unique G  $\implies$   $\exists X$  fs ms. class G Object = Some (X,fs,ms)"`  
`<proof>`

**lemma** `is_class_Object [simp]: "ws_prog G  $\implies$  is_class G Object"`  
`<proof>`

**lemma** `is_class_xcpt [simp]: "ws_prog G  $\implies$  is_class G (Xcpt x)"`  
`<proof>`

**lemma** `subcls1_wfD: "[|G  $\vdash$  C  $\prec$  C1D; ws_prog G|]  $\implies$  D  $\neq$  C  $\wedge$  (D, C)  $\notin$  (subcls1 G) $^+$ "`  
`<proof>`

**lemma** `wf_cdecl_supD:`  
`"!!r. [[ws_cdecl G (C,D,r); C  $\neq$  Object]]  $\implies$  is_class G D"`  
`<proof>`

**lemma** `subcls1_asym: "[|ws_prog G; (C, D)  $\in$  (subcls1 G) $^+$ |]  $\implies$  (D, C)  $\notin$  (subcls1 G) $^+$ "`  
`<proof>`

**lemma** `subcls1_irrefl: "[|ws_prog G; (C, D)  $\in$  (subcls1 G) $^+$ |]  $\implies$  C  $\neq$  D"`  
`<proof>`

**lemma** `acyclic_subcls1: "ws_prog G  $\implies$  acyclic (subcls1 G)"`  
`<proof>`

**lemma** `wf_subcls1: "ws_prog G  $\implies$  wf ((subcls1 G) $^{-1}$ )"`  
`<proof>`

**lemma subcls\_induct\_struct:**

"[|ws\_prog G; !!C.  $\forall D. (C, D) \in (\text{subcls1 } G)^+ \rightarrow P D \implies P C$ |]  $\implies P C$ "  
 (is "?A  $\implies \text{PROP } ?P \implies \_$ ")  
 <proof>

**lemma subcls\_induct:**

"[|wf\_prog wf\_mb G; !!C.  $\forall D. (C, D) \in (\text{subcls1 } G)^+ \rightarrow P D \implies P C$ |]  $\implies P C$ "  
 (is "?A  $\implies \text{PROP } ?P \implies \_$ ")  
 <proof>

**lemma subcls1\_induct:**

"[|is\_class G C; wf\_prog wf\_mb G; P Object;  
 !!C D fs ms. [|C  $\neq$  Object; is\_class G C; class G C = Some (D,fs,ms)  $\wedge$   
 wf\_cdecl wf\_mb G (C,D,fs,ms)  $\wedge G \vdash C \prec C1D \wedge$  is\_class G D  $\wedge P D$ |]  $\implies P C$   
 |]  $\implies P C$ "  
 (is "?A  $\implies ?B \implies ?C \implies \text{PROP } ?P \implies \_$ ")  
 <proof>

**lemma subcls1\_induct\_struct:**

"[|is\_class G C; ws\_prog G; P Object;  
 !!C D fs ms. [|C  $\neq$  Object; is\_class G C; class G C = Some (D,fs,ms)  $\wedge$   
 ws\_cdecl G (C,D,fs,ms)  $\wedge G \vdash C \prec C1D \wedge$  is\_class G D  $\wedge P D$ |]  $\implies P C$   
 |]  $\implies P C$ "  
 (is "?A  $\implies ?B \implies ?C \implies \text{PROP } ?P \implies \_$ ")  
 <proof>

**lemmas method\_rec = wf\_subcls1 [THEN [2] method\_rec\_lemma]**

**lemmas fields\_rec = wf\_subcls1 [THEN [2] fields\_rec\_lemma]**

**lemma field\_rec:** "[|class G C = Some (D, fs, ms); ws\_prog G|]  
 $\implies \text{field } (G, C) =$   
 (if C = Object then Map.empty else field (G, D)) ++  
 map\_of (map ( $\lambda(s, f). (s, C, f)$ ) fs)"  
 <proof>

**lemma method\_Object [simp]:**

"method (G, Object) sig = Some (D, mh, code)  $\implies \text{ws\_prog } G \implies D = \text{Object}$ "  
 <proof>

**lemma fields\_Object [simp]:** "[|((vn, C), T)  $\in \text{set } (\text{fields } (G, \text{Object}))$ ; ws\_prog G|]  
 $\implies C = \text{Object}$ "  
 <proof>

**lemma subcls\_C\_Object:** "[|is\_class G C; ws\_prog G|]  $\implies G \vdash C \preceq C \text{ Object}$ "  
 <proof>

**lemma is\_type\_rTI:** "wf\_mhead G sig rT  $\implies \text{is\_type } G \text{ rT}$ "  
 <proof>

**lemma widen\_fields\_defpl':** "[|is\_class G C; ws\_prog G|]  $\implies$   
 $\forall ((fn, fd), fT) \in \text{set } (\text{fields } (G, C)). G \vdash C \preceq C \text{ fd}$ "  
 <proof>



**lemma** widen\_fields\_defpl:

"[(fn,fd),fT] ∈ set (fields (G,C)); ws\_prog G; is\_class G C[] ==>  
G ⊢ C ≤<sub>C</sub> fd"  
⟨proof⟩

**lemma** unique\_fields:

"[is\_class G C; ws\_prog G[] ==> unique (fields (G,C))"  
⟨proof⟩

**lemma** fields\_mono\_lemma [rule\_format (no\_asm)]:

"[ws\_prog G; (C', C) ∈ (subcls1 G)\*[] ==>  
x ∈ set (fields (G,C)) --> x ∈ set (fields (G,C'))"  
⟨proof⟩

**lemma** fields\_mono:

"[map\_of (fields (G,C)) fn = Some f; G ⊢ D ≤<sub>C</sub> C; is\_class G D; ws\_prog G[]  
==> map\_of (fields (G,D)) fn = Some f"  
⟨proof⟩

**lemma** widen\_cfs\_fields:

"[field (G,C) fn = Some (fd, fT); G ⊢ D ≤<sub>C</sub> C; ws\_prog G[] ==>  
map\_of (fields (G,D)) (fn, fd) = Some fT"  
⟨proof⟩

**lemma** method\_wf\_mdecl [rule\_format (no\_asm)]:

"wf\_prog wf\_mb G ==> is\_class G C ==>  
method (G,C) sig = Some (md,mh,m)  
--> G ⊢ C ≤<sub>C</sub> md ∧ wf\_mdecl wf\_mb G md (sig,(mh,m))"  
⟨proof⟩

**lemma** method\_wf\_mhead [rule\_format (no\_asm)]:

"ws\_prog G ==> is\_class G C ==>  
method (G,C) sig = Some (md,rT,mb)  
--> G ⊢ C ≤<sub>C</sub> md ∧ wf\_mhead G sig rT"  
⟨proof⟩

**lemma** subcls\_widen\_methd [rule\_format (no\_asm)]:

"[G ⊢ T' ≤<sub>C</sub> T; wf\_prog wf\_mb G[] ==>  
∀ D rT b. method (G,T) sig = Some (D,rT ,b) -->  
(∃ D' rT' b'. method (G,T') sig = Some (D',rT',b')) ∧ G ⊢ D' ≤<sub>C</sub> D ∧ G ⊢ rT' ≤<sub>rT</sub> rT"  
⟨proof⟩

**lemma** subtype\_widen\_methd:

"[ G ⊢ C ≤<sub>C</sub> D; wf\_prog wf\_mb G;  
method (G,D) sig = Some (md, rT, b) []  
==> ∃ mD' rT' b'. method (G,C) sig = Some(mD',rT',b') ∧ G ⊢ rT' ≤<sub>rT</sub> rT"  
⟨proof⟩

**lemma** method\_in\_md [rule\_format (no\_asm)]:

"ws\_prog G ==> is\_class G C ==> ∀ D. method (G,C) sig = Some(D,mh,code)

```

--> is_class G D ∧ method (G,D) sig = Some(D,mh,code)"
⟨proof⟩

```

```

lemma method_in_md_struct [rule_format (no_asm)]:
  "ws_prog G ==> is_class G C ==> ∀D. method (G,C) sig = Some(D,mh,code)
  --> is_class G D ∧ method (G,D) sig = Some(D,mh,code)"
⟨proof⟩

```

```

lemma fields_in_fd [rule_format (no_asm)]: "[ wf_prog wf_mb G; is_class G C ]
  ==> ∀ vn D T. (((vn,D),T) ∈ set (fields (G,C)))
  → (is_class G D ∧ (((vn,D),T) ∈ set (fields (G,D))))"
⟨proof⟩

```

```

lemma field_in_fd [rule_format (no_asm)]: "[ wf_prog wf_mb G; is_class G C ]
  ==> ∀ vn D T. (field (G,C) vn = Some(D,T))
  → is_class G D ∧ field (G,D) vn = Some(D,T)"
⟨proof⟩

```

```

lemma widen_methd:
  "[| method (G,C) sig = Some (md,rT,b); wf_prog wf_mb G; G⊢T''⊆C C |]
  ==> ∃md' rT' b'. method (G,T'') sig = Some (md',rT',b') ∧ G⊢rT'⊆rT"
⟨proof⟩

```

```

lemma widen_field: "[ (field (G,C) fn) = Some (fd, fT); wf_prog wf_mb G; is_class G C ]
  ==> G⊢C⊆C fd"
⟨proof⟩

```

```

lemma Call_lemma:
  "[| method (G,C) sig = Some (md,rT,b); G⊢T''⊆C C; wf_prog wf_mb G;
    class G C = Some y |] ==> ∃T' rT' b. method (G,T'') sig = Some (T',rT',b) ∧
    G⊢rT'⊆rT ∧ G⊢T''⊆C T' ∧ wf_mhead G sig rT' ∧ wf_mb G T' (sig,rT',b)"
⟨proof⟩

```

```

lemma fields_is_type_lemma [rule_format (no_asm)]:
  "[| is_class G C; ws_prog G |] ==>
  ∀f∈set (fields (G,C)). is_type G (snd f)"
⟨proof⟩

```

```

lemma fields_is_type:
  "[| map_of (fields (G,C)) fn = Some f; ws_prog G; is_class G C |] ==>
  is_type G f"
⟨proof⟩

```

```

lemma field_is_type: "[ ws_prog G; is_class G C; field (G, C) fn = Some (fd, fT) ]
  ==> is_type G fT"
⟨proof⟩

```

```

lemma methd:

```

```

"[[ ws_prog G; (C,S,fs,mdecls) ∈ set G; (sig,rT,code) ∈ set mdecls ]]
==> method (G,C) sig = Some(C,rT,code) ∧ is_class G C"
⟨proof⟩

```

```

lemma wf_mb'E:
  "[ wf_prog wf_mb G; ∧ C S fs ms m. [(C,S,fs,ms) ∈ set G; m ∈ set ms] ==> wf_mb' G C m ]
  ==> wf_prog wf_mb' G"
  ⟨proof⟩

```

```

lemma fst_mono: "A ⊆ B ==> fst ` A ⊆ fst ` B" ⟨proof⟩

```

```

lemma wf_syscls:
  "set SystemClasses ⊆ set G ==> wf_syscls G"
  ⟨proof⟩

```

```

end

```

## 2.10 Well-typedness Constraints

theory WellType imports Term WellForm begin

the formulation of well-typedness of method calls given below (as well as the Java Specification 1.0) is a little too restrictive: It does not allow methods of class Object to be called upon references of interface type.

**simplifications:**

- the type rules include all static checks on expressions and statements, e.g. definedness of names (of parameters, locals, fields, methods)

local variables, including method parameters and This:

```

type_synonym lenv = "vname → ty"
type_synonym 'c env = "'c prog × lenv"

```

```

abbreviation (input)
  prg :: "'c env => 'c prog"
  where "prg == fst"

```

```

abbreviation (input)
  localT :: "'c env => (vname → ty)"
  where "localT == snd"

```

```

definition more_spec :: "'c prog ⇒ (ty × 'x) × ty list ⇒ (ty × 'x) × ty list ⇒ bool"
  where "more_spec G == λ((d,h),pTs). λ((d',h'),pTs'). G ⊢ d ≤ d' ∧
    list_all2 (λT T'. G ⊢ T ≤ T') pTs pTs'"

```

```

definition appl_methds :: "'c prog ⇒ cname ⇒ sig ⇒ ((ty × ty) × ty list) set"
  — applicable methods, cf. 15.11.2.1

```

```

where "appl_methds G C ==  $\lambda(mn, pTs).$ 
      {((Class md,rT),pTs') | md rT mb pTs'.
       method (G,C) (mn, pTs') = Some (md,rT,mb)  $\wedge$ 
       list_all2 ( $\lambda T T'. G \vdash T \preceq T'$ ) pTs pTs'}"

definition max_spec :: "'c prog  $\Rightarrow$  cname  $\Rightarrow$  sig  $\Rightarrow$  ((ty  $\times$  ty)  $\times$  ty list) set"
  — maximally specific methods, cf. 15.11.2.2
  where "max_spec G C sig == {m. m  $\in$  appl_methds G C sig  $\wedge$ 
      ( $\forall m' \in$  appl_methds G C sig.
       more_spec G m' m  $\rightarrow$  m' = m)}"

lemma max_spec2appl_meths:
  "x  $\in$  max_spec G C sig  $\Rightarrow$  x  $\in$  appl_methds G C sig"
  <proof>

lemma appl_methsD:
  "((md,rT),pTs')  $\in$  appl_methds G C (mn, pTs)  $\Rightarrow$ 
    $\exists D b. md = \text{Class } D \wedge \text{method } (G,C) (mn, pTs') = \text{Some } (D,rT,b)$ 
    $\wedge \text{list\_all2 } (\lambda T T'. G \vdash T \preceq T') pTs pTs'$ "
  <proof>

lemmas max_spec2mheads = insertI1 [THEN [2] equalityD2 [THEN subsetD],
  THEN max_spec2appl_meths, THEN appl_methsD]

primrec typeof :: "(loc  $\Rightarrow$  ty option)  $\Rightarrow$  val  $\Rightarrow$  ty option"
  where
    "typeof dt Unit      = Some (PrimT Void)"
  | "typeof dt Null      = Some NT"
  | "typeof dt (Bool b)  = Some (PrimT Boolean)"
  | "typeof dt (Intg i)  = Some (PrimT Integer)"
  | "typeof dt (Addr a)  = dt a"

lemma is_type_typeof [rule_format (no_asm), simp]:
  "( $\forall a. v \neq \text{Addr } a \rightarrow (\exists T. \text{typeof } t v = \text{Some } T \wedge \text{is\_type } G T)$ )"
  <proof>

lemma typeof_empty_is_type [rule_format (no_asm)]:
  "typeof ( $\lambda a. \text{None}$ ) v = Some T  $\rightarrow$  is_type G T"
  <proof>

lemma typeof_default_val: " $\exists T. (\text{typeof } dt (\text{default\_val } ty) = \text{Some } T) \wedge G \vdash T \preceq ty$ "
  <proof>

type_synonym
  java_mb = "vname list  $\times$  (vname  $\times$  ty) list  $\times$  stmt  $\times$  expr"
  — method body with parameter names, local variables, block, result expression.
  — local variables might include This, which is hidden anyway

inductive
  ty_expr :: "'c env  $\Rightarrow$  expr  $\Rightarrow$  ty  $\Rightarrow$  bool" (<_  $\vdash$  _ :: _> [51, 51, 51] 50)
  and ty_exprs :: "'c env  $\Rightarrow$  expr list  $\Rightarrow$  ty list  $\Rightarrow$  bool" (<_  $\vdash$  _ [::] _> [51, 51, 51] 50)
  and wt_stmt :: "'c env  $\Rightarrow$  stmt  $\Rightarrow$  bool" (<_  $\vdash$  _  $\sqrt{\phantom{x}}$ > [51, 51] 50)

```

where

```

NewC: "[| is_class (prg E) C |] ==>
      E⊢NewC C::Class C" — cf. 15.8

— cf. 15.15
/ Cast: "[| E⊢e::C; is_class (prg E) D;
          prg E⊢C⊆? Class D |] ==>
      E⊢Cast D e::Class D"

— cf. 15.7.1
/ Lit: "[| typeof (λv. None) x = Some T |] ==>
      E⊢Lit x::T"

— cf. 15.13.1
/ LAcc: "[| localT E v = Some T; is_type (prg E) T |] ==>
      E⊢LAcc v::T"

/ BinOp: "[| E⊢e1::T;
            E⊢e2::T;
            if bop = Eq then T' = PrimT Boolean
            else T' = T ∧ T = PrimT Integer |] ==>
      E⊢BinOp bop e1 e2::T'"

— cf. 15.25, 15.25.1
/ LAss: "[| v ~= This;
          E⊢LAcc v::T;
          E⊢e::T';
          prg E⊢T'⊆T |] ==>
      E⊢v::e::T'"

— cf. 15.10.1
/ FAcc: "[| E⊢a::Class C;
          field (prg E,C) fn = Some (fd,fT) |] ==>
      E⊢{fd}a..fn::fT"

— cf. 15.25, 15.25.1
/ FAss: "[| E⊢{fd}a..fn::T;
          E⊢v      ::T';
          prg E⊢T'⊆T |] ==>
      E⊢{fd}a..fn:=v::T'"

— cf. 15.11.1, 15.11.2, 15.11.3
/ Call: "[| E⊢a::Class C;
          E⊢ps[::]pTs;
          max_spec (prg E) C (mn, pTs) = {(⟨md,rT⟩,pTs')} |] ==>
      E⊢{C}a..mn({pTs'}ps)::rT"

— well-typed expression lists

— cf. 15.11.???
/ Nil: "E⊢[][::][]"

```

```

— cf. 15.11.???
| Cons:"[| E⊢e::T;
      E⊢es[::]Ts |] ==>
      E⊢e#es[::]T#Ts"

— well-typed statements

| Skip:"E⊢Skip√"

| Expr:"[| E⊢e::T |] ==>
      E⊢Expr e√"

| Comp:"[| E⊢s1√;
      E⊢s2√ |] ==>
      E⊢s1;; s2√"

— cf. 14.8
| Cond:"[| E⊢e::PrimT Boolean;
      E⊢s1√;
      E⊢s2√ |] ==>
      E⊢If(e) s1 Else s2√"

— cf. 14.10
| Loop:"[| E⊢e::PrimT Boolean;
      E⊢s√ |] ==>
      E⊢While(e) s√"

definition wf_java_mdecl :: "'c prog => cname => java_mb mdecl => bool" where
"wf_java_mdecl G C == λ((mn,pTs),rT,(pns,lvars,blk,res)).
  length pTs = length pns ∧
  distinct pns ∧
  unique lvars ∧
  This ∉ set pns ∧ This ∉ set (map fst lvars) ∧
  (∀pn∈set pns. map_of lvars pn = None) ∧
  (∀(vn,T)∈set lvars. is_type G T) &
  (let E = (G,(map_of lvars)(pns[↦]pTs, This↦Class C)) in
  E⊢blk√ ∧ (∃T. E⊢res::T ∧ G⊢T⊑rT))"

abbreviation "wf_java_prog == wf_prog wf_java_mdecl"

lemma wf_java_prog_wf_java_mdecl: "[[
  wf_java_prog G; (C, D, fds, mths) ∈ set G; jmdcl ∈ set mths ]
  ==> wf_java_mdecl G C jmdcl"
<proof>

lemma wt_is_type: "(E⊢e::T → ws_prog (prg E) → is_type (prg E) T) ∧
  (E⊢es[::]Ts → ws_prog (prg E) → Ball (set Ts) (is_type (prg E))) ∧
  (E⊢c √ → True)"
<proof>

lemmas ty_expr_is_type = wt_is_type [THEN conjunct1, THEN mp, rule_format]

```

```

lemma expr_class_is_class: "
  [[ws_prog (prg E); E ⊢ e :: Class C]] ⇒ is_class (prg E) C"
  ⟨proof⟩

```

```
end
```

## 2.11 Operational Evaluation (big step) Semantics

```
theory Eval imports State WellType begin
```

— Auxiliary notions

```

definition fits :: "java_mb prog ⇒ state ⇒ val ⇒ ty ⇒ bool" (<_,_>fits_>[61,61,61,61]60)
where
  "G,s⊢a' fits T ≡ case T of PrimT T' ⇒ False | RefT T' ⇒ a'=Null ∨ G⊢obj_ty(lookup_obj
s a')≤T"

```

```

definition catch :: "java_mb prog ⇒ xstate ⇒ cname ⇒ bool" (<_,_>catch_>[61,61,61]60)
where
  "G,s⊢catch C ≡ case abrupt s of None ⇒ False | Some a ⇒ G,store s⊢ a fits Class C"

```

```

definition lupd :: "vname ⇒ val ⇒ state ⇒ state" (<lupd'(_↦_)>[10,10]1000) where
  "lupd vn v ≡ λ (hp,loc). (hp, (loc(vn↦v)))"

```

```

definition new_xcpt_var :: "vname ⇒ xstate ⇒ xstate" where
  "new_xcpt_var vn ≡ λ(x,s). Norm (lupd(vn↦the x) s)"

```

— Evaluation relations

```

inductive
  eval :: "[java_mb prog,xstate,expr,val,xstate] => bool "
    (<_ ⊢ _ ->_> [51,82,60,82,82] 81)
  and evals :: "[java_mb prog,xstate,expr list,
    val list,xstate] => bool "
    (<_ ⊢ _ ->_> [51,82,60,51,82] 81)
  and exec :: "[java_mb prog,xstate,stmt, xstate] => bool "
    (<_ ⊢ _ ->_> [51,82,60,82] 81)
  for G :: "java_mb prog"
where

```

— evaluation of expressions

$XcptE: "G ⊢ (Some xc, s) -e>undefined-> (Some xc, s)"$  — cf. 15.5

— cf. 15.8.1

```

| NewC: "[| h = heap s; (a,x) = new_Addr h;
  h' = h(a↦(C,init_vars (fields (G,C)))) |] ==>
  G⊢Norm s -NewC C>Addr a-> c_hupd h' (x,s)"

```

```

— cf. 15.15
| Cast: "[| G⊢Norm s0 -e>v-> (x1,s1);
          x2 = raise_if (¬ cast_ok G C (heap s1) v) ClassCast x1 |] ==>
          G⊢Norm s0 -Cast C e>v-> (x2,s1)"

— cf. 15.7.1
| Lit: "G⊢Norm s -Lit v>v-> Norm s"

| BinOp: "[| G⊢Norm s -e1>v1-> s1;
             G⊢s1 -e2>v2-> s2;
             v = (case bop of Eq => Bool (v1 = v2)
                  | Add => Intg (the_Intg v1 + the_Intg v2)) |] ==>
             G⊢Norm s -BinOp bop e1 e2>v-> s2"

— cf. 15.13.1, 15.2
| LAcc: "G⊢Norm s -LAcc v>the (locals s v)-> Norm s"

— cf. 15.25.1
| LAss: "[| G⊢Norm s -e>v-> (x,(h,l));
            l' = (if x = None then l(va↦v) else l) |] ==>
            G⊢Norm s -va::e>v-> (x,(h,l'))"

— cf. 15.10.1, 15.2
| FAcc: "[| G⊢Norm s0 -e>a'-> (x1,s1);
            v = the (snd (the (heap s1 (the_Addr a')))) (fn,T)) |] ==>
            G⊢Norm s0 -{T}e..fn>v-> (np a' x1,s1)"

— cf. 15.25.1
| FAss: "[| G⊢      Norm s0 -e1>a'-> (x1,s1); a = the_Addr a';
            G⊢(np a' x1,s1) -e2>v-> (x2,s2);
            h = heap s2; (c,fs) = the (h a);
            h' = h(a↦(c,(fs((fn,T)↦v)))) |] ==>
            G⊢Norm s0 -{T}e1..fn:=e2>v-> c_hupd h' (x2,s2)"

— cf. 15.11.4.1, 15.11.4.2, 15.11.4.4, 15.11.4.5, 14.15
| Call: "[| G⊢Norm s0 -e>a'-> s1; a = the_Addr a';
            G⊢s1 -ps[>]pvs-> (x,(h,l)); dynT = fst (the (h a));
            (md,rT,pns,lvars,blk,res) = the (method (G,dynT) (mn,pTs));
            G⊢(np a' x,(h,(init_vars lvars)(pns[↦]pvs, This↦a')) -blk-> s3;
            G⊢ s3 -res>v-> (x4,s4) |] ==>
            G⊢Norm s0 -{C}e..mn({pTs}ps)>v-> (x4,(heap s4,l))"

— evaluation of expression lists

— cf. 15.5
| XcptEs: "G⊢(Some xc,s) -e[>]undefined-> (Some xc,s)"

— cf. 15.11.???
| Nil: "G⊢Norm s0 -[] [>] []-> Norm s0"

— cf. 15.6.4
| Cons: "[| G⊢Norm s0 -e > v-> s1;
            G⊢      s1 -es[>]vs-> s2 |] ==>

```



```

    G⊢Norm s0 -e#es[>]v#vs-> s2"

— execution of statements

— cf. 14.1
| XcptS: "G⊢(Some xc,s) -c-> (Some xc,s)"

— cf. 14.5
| Skip: "G⊢Norm s -Skip-> Norm s"

— cf. 14.7
| Expr: "[| G⊢Norm s0 -e>v-> s1 |] ==>
    G⊢Norm s0 -Expr e-> s1"

— cf. 14.2
| Comp: "[| G⊢Norm s0 -c1-> s1;
    G⊢      s1 -c2-> s2|] ==>
    G⊢Norm s0 -c1;; c2-> s2"

— cf. 14.8.2
| Cond: "[| G⊢Norm s0 -e>v-> s1;
    G⊢ s1 -(if the_Bool v then c1 else c2)-> s2|] ==>
    G⊢Norm s0 -If(e) c1 Else c2-> s2"

— cf. 14.10, 14.10.1
| LoopF: "[| G⊢Norm s0 -e>v-> s1; ¬the_Bool v |] ==>
    G⊢Norm s0 -While(e) c-> s1"
| LoopT: "[| G⊢Norm s0 -e>v-> s1; the_Bool v;
    G⊢s1 -c-> s2; G⊢s2 -While(e) c-> s3 |] ==>
    G⊢Norm s0 -While(e) c-> s3"

lemmas eval_evals_exec_induct = eval_evals_exec.induct [split_format (complete)]

lemma NewCI: "[|new_Addr (heap s) = (a,x);
    s' = c_hupd ((heap s)(a↦(C,init_vars (fields (G,C)))) (x,s))|] ==>
    G⊢Norm s -NewC C>Addr a-> s'"
⟨proof⟩

lemma eval_evals_exec_no_xcpt:
  "!!s s'. (G⊢(x,s) -e > v -> (x',s') --> x'=None --> x=None) ∧
    (G⊢(x,s) -es[>]vs-> (x',s') --> x'=None --> x=None) ∧
    (G⊢(x,s) -c -> (x',s') --> x'=None --> x=None)"
⟨proof⟩

lemma eval_no_xcpt: "G⊢(x,s) -e>v-> (None,s') ==> x=None"
⟨proof⟩

lemma evals_no_xcpt: "G⊢(x,s) -e[>]v-> (None,s') ==> x=None"
⟨proof⟩

lemma exec_no_xcpt: "G ⊢ (x, s) -c-> (None, s')
  ==> x = None"

```

⟨proof⟩

**lemma** eval\_evals\_exec\_xcpt:

```
"!!s s'. (G⊢(x,s) -e> v -> (x',s')) --> x=Some xc --> x'=Some xc ∧ s'=s) ∧
          (G⊢(x,s) -es[>]vs-> (x',s')) --> x=Some xc --> x'=Some xc ∧ s'=s) ∧
          (G⊢(x,s) -c          -> (x',s')) --> x=Some xc --> x'=Some xc ∧ s'=s)"
```

⟨proof⟩

**lemma** eval\_xcpt: "G⊢(Some xc,s) -e>v-> (x',s') ==> x'=Some xc ∧ s'=s"

⟨proof⟩

**lemma** exec\_xcpt: "G⊢(Some xc,s) -s0-> (x',s') ==> x'=Some xc ∧ s'=s"

⟨proof⟩

**lemma** eval\_LAcc\_code: "G⊢Norm (h, l) -LAcc v>the (l v)-> Norm (h, l)"

⟨proof⟩

**lemma** eval\_Call\_code:

```
"[| G⊢Norm s0 -e>a'-> s1; a = the_Addr a';
   G⊢s1 -ps[>]pvs-> (x,(h,l)); dynT = fst (the (h a));
   (md,rT,pns,lvars,blk,res) = the (method (G,dynT) (mn,pTs));
   G⊢(np a' x,(h,(init_vars lvars)(pns[>]pvs, This→a')) -blk-> s3;
   G⊢ s3 -res>v -> (x4,(h4, l4)) |] ==>
   G⊢Norm s0 -{C}e..mn({pTs}ps)>v-> (x4,(h4,l))"
```

⟨proof⟩

**lemmas** [code\_pred\_intro] = XcptE NewC Cast Lit BinOp

**lemmas** [code\_pred\_intro LAcc\_code] = eval\_LAcc\_code

**lemmas** [code\_pred\_intro] = LAss FAcc FAss

**lemmas** [code\_pred\_intro Call\_code] = eval\_Call\_code

**lemmas** [code\_pred\_intro] = XcptEs Nil Cons XcptS Skip Expr Comp Cond LoopF

**lemmas** [code\_pred\_intro LoopT\_code] = LoopT

**code\_pred**

```
(modes:
  eval: i ⇒ i ⇒ i ⇒ o ⇒ o ⇒ bool
and
  evals: i ⇒ i ⇒ i ⇒ o ⇒ o ⇒ bool
and
  exec: i ⇒ i ⇒ i ⇒ o ⇒ bool)
eval
```

⟨proof⟩

**end**

**theory** Exceptions **imports** State **begin**

a new, blank object with default values in all fields:

**definition** blank :: "'c prog ⇒ cname ⇒ obj" **where**

```
"blank G C ≡ (C,init_vars (fields(G,C)))"
```

```

definition start_heap :: "'c prog  $\Rightarrow$  aheap" where
  "start_heap G  $\equiv$  Map.empty (XcptRef NullPointer  $\mapsto$  blank G (Xcpt NullPointer),
    XcptRef ClassCast  $\mapsto$  blank G (Xcpt ClassCast),
    XcptRef OutOfMemory  $\mapsto$  blank G (Xcpt OutOfMemory))"

```

**abbreviation**

```

cname_of :: "aheap  $\Rightarrow$  val  $\Rightarrow$  cname"
where "cname_of hp v == fst (the (hp (the_Addr v)))"

```

```

definition preallocated :: "aheap  $\Rightarrow$  bool" where
  "preallocated hp  $\equiv \forall x. \exists fs. hp (XcptRef x) = Some (Xcpt x, fs)"$ 
```

```

lemma preallocatedD:
  "preallocated hp  $\implies \exists fs. hp (XcptRef x) = Some (Xcpt x, fs)"
  <proof>$ 
```

```

lemma preallocatedE [elim?]:
  "preallocated hp  $\implies (\bigwedge fs. hp (XcptRef x) = Some (Xcpt x, fs) \implies P hp) \implies P hp"$ 
  <proof>

```

```

lemma cname_of_xcp:
  "raise_if b x None = Some xcp  $\implies$  preallocated hp
 $\implies$  cname_of (hp::aheap) xcp = Xcpt x"
  <proof>

```

```

lemma preallocated_start:
  "preallocated (start_heap G)"
  <proof>

```

**end**

## 2.12 Conformity Relations for Type Soundness Proof

```

theory Conform imports State WellType Exceptions begin

```

```

type_synonym 'c env' = "'c prog  $\times$  (vname  $\rightarrow$  ty)" — same as env of WellType.thy

```

```

definition hext :: "aheap  $\Rightarrow$  aheap  $\Rightarrow$  bool" ( $\langle\_ \leq | \_ \rangle$  [51,51] 50) where
  " $h \leq | h' == \forall a C fs. h a = Some (C, fs) \rightarrow (\exists fs'. h' a = Some (C, fs'))"$ 

```

```

definition conf :: "'c prog  $\Rightarrow$  aheap  $\Rightarrow$  val  $\Rightarrow$  ty  $\Rightarrow$  bool"
  ( $\langle\_ , \_ \vdash \_ :: \leq \_ \rangle$  [51,51,51,51] 50) where
  " $G, h \vdash v :: \leq T == \exists T'. \text{typeof} (\text{map\_option obj\_ty } o h) v = Some T' \wedge G \vdash T' \leq T"$ 

```

```

definition lconf :: "'c prog  $\Rightarrow$  aheap  $\Rightarrow$  ('a  $\rightarrow$  val)  $\Rightarrow$  ('a  $\rightarrow$  ty)  $\Rightarrow$  bool"
  ( $\langle\_ , \_ \vdash \_ [:: \leq] \_ \rangle$  [51,51,51,51] 50) where
  " $G, h \vdash vs [:: \leq] Ts == \forall n T. Ts n = Some T \rightarrow (\exists v. vs n = Some v \wedge G, h \vdash v :: \leq T)"$ 

```

```

definition oconf :: "'c prog  $\Rightarrow$  aheap  $\Rightarrow$  obj  $\Rightarrow$  bool" ( $\langle\_ , \_ \vdash \_ \sqrt{\phantom{x}} \rangle$  [51,51,51] 50) where

```

"G,h $\vdash$ obj  $\surd$  == G,h $\vdash$ snd obj[: $\preceq$ ]map\_of (fields (G,fst obj))"

**definition** hconf :: "'c prog => aheap => bool" (<\_  $\vdash$  h \_  $\surd$ > [51,51] 50) where  
 "G $\vdash$ h h  $\surd$  ==  $\forall$  a obj. h a = Some obj --> G,h $\vdash$ obj  $\surd$ "

**definition** xconf :: "aheap  $\Rightarrow$  val option  $\Rightarrow$  bool" where  
 "xconf hp vo == preallocated hp  $\wedge$  ( $\forall$  v. (vo = Some v)  $\longrightarrow$  ( $\exists$  xc. v = (Addr (XcptRef xc))))"

**definition** conforms :: "xstate => java\_mb env' => bool" (<\_ :: $\preceq$  \_> [51,51] 50) where  
 "s:: $\preceq$ E == prg E $\vdash$ h heap (store s)  $\surd$   $\wedge$   
           prg E,heap (store s) $\vdash$ locals (store s)[: $\preceq$ ]localT E  $\wedge$   
           xconf (heap (store s)) (abrupt s)"

### 2.12.1 hext

**lemma** hextI:  
 "  $\forall$  a C fs . h a = Some (C,fs) -->  
   ( $\exists$  fs'. h' a = Some (C,fs')) ==> h $\leq$ |h' "  
 <proof>

**lemma** hext\_objD: "[|h $\leq$ |h'; h a = Some (C,fs) |] ==>  $\exists$  fs'. h' a = Some (C,fs')"  
 <proof>

**lemma** hext\_refl [simp]: "h $\leq$ |h"  
 <proof>

**lemma** hext\_new [simp]: "h a = None ==> h $\leq$ |h(a $\mapsto$ x)"  
 <proof>

**lemma** hext\_trans: "[|h $\leq$ |h'; h' $\leq$ |h''|] ==> h $\leq$ |h''"  
 <proof>

**lemma** hext\_upd\_obj: "h a = Some (C,fs) ==> h $\leq$ |h(a $\mapsto$ (C,fs'))"  
 <proof>

### 2.12.2 conf

**lemma** conf\_Null [simp]: "G,h $\vdash$ Null:: $\preceq$ T = G $\vdash$ RefT NullT $\preceq$ T"  
 <proof>

**lemma** conf\_litval [rule\_format (no\_asm), simp]:  
 "typeof ( $\lambda$ v. None) v = Some T --> G,h $\vdash$ v:: $\preceq$ T"  
 <proof>

**lemma** conf\_AddrI: "[|h a = Some obj; G $\vdash$ obj\_ty obj $\preceq$ T|] ==> G,h $\vdash$ Addr a:: $\preceq$ T"  
 <proof>

**lemma** conf\_obj\_AddrI: "[|h a = Some (C,fs); G $\vdash$ C $\preceq$ C D|] ==> G,h $\vdash$ Addr a:: $\preceq$  Class D"  
 <proof>

**lemma** defval\_conf [rule\_format (no\_asm)]:  
 "is\_type G T --> G,h $\vdash$ default\_val T:: $\preceq$ T"  
 <proof>

**lemma** *conf\_upd\_obj*:

" $\exists a = \text{Some } (C, fs) \implies (G, h(a \mapsto (C, fs'))) \vdash x :: \preceq T = (G, h \vdash x :: \preceq T)$ "  
 ⟨proof⟩

**lemma** *conf\_widen [rule\_format (no\_asm)]*:

" $\text{wf\_prog wf\_mb } G \implies G, h \vdash x :: \preceq T \dashv\vdash G \vdash T \preceq T' \dashv\vdash G, h \vdash x :: \preceq T'$ "  
 ⟨proof⟩

**lemma** *conf\_hext [rule\_format (no\_asm)]*: " $h \leq |h' \implies G, h \vdash v :: \preceq T \dashv\vdash G, h' \vdash v :: \preceq T$ "

⟨proof⟩

**lemma** *new\_locD*: " $[|h a = \text{None}; G, h \vdash \text{Addr } t :: \preceq T|] \implies t \neq a$ "

⟨proof⟩

**lemma** *conf\_RefTD [rule\_format]*:

" $G, h \vdash a' :: \preceq \text{RefT } T \implies a' = \text{Null} \vee$   
 $(\exists a \text{ obj } T'. a' = \text{Addr } a \wedge h a = \text{Some obj} \wedge \text{obj\_ty obj} = T' \wedge G \vdash T' \preceq \text{RefT } T)$ "  
 ⟨proof⟩

**lemma** *conf\_NullTD*: " $G, h \vdash a' :: \preceq \text{RefT NullT} \implies a' = \text{Null}$ "

⟨proof⟩

**lemma** *non\_npD*: " $[|a' \neq \text{Null}; G, h \vdash a' :: \preceq \text{RefT } t|] \implies$

$\exists a C fs. a' = \text{Addr } a \wedge h a = \text{Some } (C, fs) \wedge G \vdash \text{Class } C \preceq \text{RefT } t$ "  
 ⟨proof⟩

**lemma** *non\_np\_objD*: " $!!G. [|a' \neq \text{Null}; G, h \vdash a' :: \preceq \text{Class } C|] \implies$

$(\exists a C' fs. a' = \text{Addr } a \wedge h a = \text{Some } (C', fs) \wedge G \vdash C' \preceq C C)$ "  
 ⟨proof⟩

**lemma** *non\_np\_objD' [rule\_format (no\_asm)]*:

" $a' \neq \text{Null} \implies \text{wf\_prog wf\_mb } G \implies G, h \vdash a' :: \preceq \text{RefT } t \dashv\vdash$   
 $(\exists a C fs. a' = \text{Addr } a \wedge h a = \text{Some } (C, fs) \wedge G \vdash \text{Class } C \preceq \text{RefT } t)$ "  
 ⟨proof⟩

**lemma** *conf\_list\_gext\_widen [rule\_format (no\_asm)]*:

" $\text{wf\_prog wf\_mb } G \implies \forall Ts Ts'. \text{list\_all2 } (\text{conf } G h) \text{ vs } Ts \dashv\vdash$   
 $\text{list\_all2 } (\lambda T T'. G \vdash T \preceq T') Ts Ts' \dashv\vdash \text{list\_all2 } (\text{conf } G h) \text{ vs } Ts'$ "  
 ⟨proof⟩

### 2.12.3 lconf

**lemma** *lconfD*: " $[| G, h \vdash \text{vs} [:: \preceq] Ts; Ts n = \text{Some } T |] \implies G, h \vdash (\text{the } (vs n)) :: \preceq T$ "

⟨proof⟩

**lemma** *lconf\_hext [elim]*: " $[| G, h \vdash l [:: \preceq] L; h \leq |h' |] \implies G, h' \vdash l [:: \preceq] L$ "

⟨proof⟩

**lemma** *lconf\_upd*: " $!!X. [| G, h \vdash l [:: \preceq] lT;$

$G, h \vdash v :: \preceq T; lT va = \text{Some } T |] \implies G, h \vdash l(va \mapsto v) [:: \preceq] lT$ "  
 ⟨proof⟩

**lemma** *lconf\_init\_vars\_lemma [rule\_format (no\_asm)]*:

```

"∀x. P x --> R (dv x) x ==> (∀x. map_of fs f = Some x --> P x) -->
(∀T. map_of fs f = Some T -->
(∃v. map_of (map (λ(f,ft). (f, dv ft)) fs) f = Some v ∧ R v T))"
⟨proof⟩

```

```

lemma lconf_init_vars [intro!]:
"∀n. ∀T. map_of fs n = Some T --> is_type G T ==> G, h ⊢ init_vars fs [::≼] map_of fs"
⟨proof⟩

```

```

lemma lconf_ext: "[|G, s ⊢ l [::≼] L; G, s ⊢ v ::≼ T|] ==> G, s ⊢ l (vn ↦ v) [::≼] L (vn ↦ T)"
⟨proof⟩

```

```

lemma lconf_ext_list [rule_format (no_asm)]:
"G, h ⊢ l [::≼] L ==> ∀ vs Ts. distinct vns --> length Ts = length vns -->
list_all2 (λv T. G, h ⊢ v ::≼ T) vs Ts --> G, h ⊢ l (vns [↦] vs) [::≼] L (vns [↦] Ts)"
⟨proof⟩

```

```

lemma lconf_restr: "[|lT vn = None; G, h ⊢ l [::≼] lT (vn ↦ T)|] ==> G, h ⊢ l [::≼] lT"
⟨proof⟩

```

#### 2.12.4 oconf

```

lemma oconf_hext: "G, h ⊢ obj √ ==> h ≤ |h' ==> G, h' ⊢ obj √"
⟨proof⟩

```

```

lemma oconf_obj: "G, h ⊢ (C, fs) √ =
(∀T f. map_of (fields (G, C)) f = Some T --> (∃v. fs f = Some v ∧ G, h ⊢ v ::≼ T))"
⟨proof⟩

```

```

lemmas oconf_objD = oconf_obj [THEN iffD1, THEN spec, THEN spec, THEN mp]

```

#### 2.12.5 hconf

```

lemma hconfD: "[|G ⊢ h h √; h a = Some obj|] ==> G, h ⊢ obj √"
⟨proof⟩

```

```

lemma hconfI: "∀ a obj. h a = Some obj --> G, h ⊢ obj √ ==> G ⊢ h h √"
⟨proof⟩

```

#### 2.12.6 xconf

```

lemma xconf_raise_if: "xconf h x ==> xconf h (raise_if b xcn x)"
⟨proof⟩

```

#### 2.12.7 conforms

```

lemma conforms_heapD: "(x, (h, l)) ::≼ (G, lT) ==> G ⊢ h h √"
⟨proof⟩

```

```

lemma conforms_localD: "(x, (h, l)) ::≼ (G, lT) ==> G, h ⊢ l [::≼] lT"
⟨proof⟩

```

```

lemma conforms_xcptD: "(x, (h, l)) ::≼ (G, lT) ==> xconf h x"
⟨proof⟩

```

**lemma conformsI:** " $[|G \vdash h \text{ h}\checkmark; G, h \vdash l[:\preceq] 1T; \text{xconf } h \text{ x}|] \implies (x, (h, l))::\preceq(G, 1T)$ "  
 $\langle \text{proof} \rangle$

**lemma conforms\_restr:** " $[|1T \text{ vn} = \text{None}; s::\preceq(G, 1T(\text{vn} \mapsto T))|] \implies s::\preceq(G, 1T)$ "  
 $\langle \text{proof} \rangle$

**lemma conforms\_xcpt\_change:** " $[|(x, (h, l))::\preceq(G, 1T); \text{xconf } h \text{ x} \longrightarrow \text{xconf } h \text{ x}'|] \implies (x', (h, l))::\preceq(G, 1T)$ "  
 $\langle \text{proof} \rangle$

**lemma preallocated\_hext:** " $[|\text{preallocated } h; h \leq |h'|] \implies \text{preallocated } h'$ "  
 $\langle \text{proof} \rangle$

**lemma xconf\_hext:** " $[|\text{xconf } h \text{ vo}; h \leq |h'|] \implies \text{xconf } h' \text{ vo}$ "  
 $\langle \text{proof} \rangle$

**lemma conforms\_hext:** " $[|(x, (h, l))::\preceq(G, 1T); h \leq |h'|; G \vdash h \text{ h}'\checkmark|] \implies (x, (h', l))::\preceq(G, 1T)$ "  
 $\langle \text{proof} \rangle$

**lemma conforms\_upd\_obj:**  
 $[|(x, (h, l))::\preceq(G, 1T); G, h(a \mapsto \text{obj}) \vdash \text{obj}\checkmark; h \leq |h(a \mapsto \text{obj})|] \implies (x, (h(a \mapsto \text{obj}), l))::\preceq(G, 1T)$   
 $\langle \text{proof} \rangle$

**lemma conforms\_upd\_local:**  
 $[|(x, (h, l))::\preceq(G, 1T); G, h \vdash v::\preceq T; 1T \text{ va} = \text{Some } T|] \implies (x, (h, l(\text{va} \mapsto v)))::\preceq(G, 1T)$   
 $\langle \text{proof} \rangle$

**end**

## 2.13 Type Safety Proof

**theory JTypeSafe imports Eval Conform begin**

**declare split\_beta [simp]**

**lemma NewC\_conforms:**  
 $[|h \text{ a} = \text{None}; (x, (h, l))::\preceq(G, 1T); \text{wf\_prog wf\_mb } G; \text{is\_class } G \text{ C}|] \implies (x, (h(a \mapsto (C, (\text{init\_vars } (\text{fields } (G, C))))), l))::\preceq(G, 1T)$   
 $\langle \text{proof} \rangle$

**lemma Cast\_conf:**  
 $[|\text{wf\_prog wf\_mb } G; G, h \vdash v::\preceq CC; G \vdash CC \preceq? \text{Class } D; \text{cast\_ok } G \text{ D } h \text{ v}|] \implies G, h \vdash v::\preceq \text{Class } D$   
 $\langle \text{proof} \rangle$

**lemma FAcc\_type\_sound:**

```
"[| wf_prog wf_mb G; field (G,C) fn = Some (fd, ft); (x,(h,l))::≤(G,lT);
  x' = None --> G,h⊢a'::≤ Class C; np a' x' = None |] ==>
  G,h⊢the (snd (the (h (the_Addr a'))) (fn, fd))::≤ft"
⟨proof⟩
```

**lemma FAss\_type\_sound:**

```
"[| wf_prog wf_mb G; a = the_Addr a'; (c, fs) = the (h a);
  (G, lT)⊢v::T'; G⊢T'≤ft;
  (G, lT)⊢aa::Class C;
  field (G,C) fn = Some (fd, ft); h''≤|h';
  x' = None --> G,h'⊢a'::≤ Class C; h'≤|h;
  Norm (h, l)::≤(G, lT); G,h⊢x::≤T'; np a' x' = None|] ==>
  h''≤|h(a↦(c,(fs((fn,fd)↦x)))) ∧
  Norm(h(a↦(c,(fs((fn,fd)↦x)))), l)::≤(G, lT) ∧
  G,h(a↦(c,(fs((fn,fd)↦x))))⊢x::≤T'"
⟨proof⟩
```

**lemma Call\_lemma2:** "[| wf\_prog wf\_mb G; list\_all2 (conf G h) pvs pTs;  
 list\_all2 (λT T'. G⊢T≤T') pTs pTs'; wf\_mhead G (mn,pTs') rT;  
 length pTs' = length pns; distinct pns;  
 Ball (set lvars) (case\_prod (λvn. is\_type G))  
 |] ==> G,h⊢(init\_vars lvars)(pns[↦]pvs)[::≤](map\_of lvars)(pns[↦]pTs')"  
 ⟨proof⟩

**lemma Call\_type\_sound:**

```
"[| wf_java_prog G; a' ≠ Null; Norm (h, l)::≤(G, lT); class G C = Some y;
  max_spec G C (mn,pTs) = {(mda,rTa),pTs'}; xc≤|xh; xh≤|h;
  list_all2 (conf G h) pvs pTs;
  (md, rT, pns, lvars, blk, res) =
    the (method (G,fst (the (h (the_Addr a')))) (mn, pTs'));
  ∀lT. (np a' None, h, (init_vars lvars)(pns[↦]pvs, This↦a'))::≤(G, lT) -->
  (G, lT)⊢blk✓ --> h≤|xi ∧ (xcptb, xi, xl)::≤(G, lT);
  ∀lT. (xcptb,xi, xl)::≤(G, lT) --> (∀T. (G, lT)⊢res::T -->
    xi≤|h' ∧ (x',h', xj)::≤(G, lT) ∧ (x' = None --> G,h'⊢v::≤T));
  G,xh⊢a'::≤ Class C
  |] ==>
  xc≤|h' ∧ (x',(h', l))::≤(G, lT) ∧ (x' = None --> G,h'⊢v::≤rTa)"
⟨proof⟩
```

```
declare if_split [split del]
declare fun_upd_apply [simp del]
declare fun_upd_same [simp]
declare wf_prog_ws_prog [simp]
```

⟨ML⟩

**theorem eval\_evals\_exec\_type\_sound:**

```
"wf_java_prog G ==>
  (G⊢(x,(h,l)) -e >v -> (x', (h',l'))) -->
```



```

(∀ lT. (x, (h, l)) :: ≤(G, lT) --> (∀ T. (G, lT) ⊢ e :: T -->
  h ≤ |h' ∧ (x', (h', l')) :: ≤(G, lT) ∧ (x' = None --> G, h' ⊢ v :: ≤ T))) ∧
(G ⊢ (x, (h, l)) -es[>] vs -> (x', (h', l')) -->
  (∀ lT. (x, (h, l)) :: ≤(G, lT) --> (∀ Ts. (G, lT) ⊢ es[:, :] Ts -->
    h ≤ |h' ∧ (x', (h', l')) :: ≤(G, lT) ∧ (x' = None --> list_all2 (λv T. G, h' ⊢ v :: ≤ T) vs Ts))))
∧
(G ⊢ (x, (h, l)) -c --> (x', (h', l')) -->
  (∀ lT. (x, (h, l)) :: ≤(G, lT) --> (G, lT) ⊢ c √ -->
    h ≤ |h' ∧ (x', (h', l')) :: ≤(G, lT)))"
⟨proof⟩

```

```

lemma eval_type_sound: "!!E s s'.
  [| wf_java_prog G; G ⊢ (x, s) -e>v -> (x', s'); (x, s) :: ≤E; E ⊢ e :: T; G = prg E |]
  ==> (x', s') :: ≤E ∧ (x' = None --> G, heap s' ⊢ v :: ≤ T) ∧ heap s ≤ | heap s'"
⟨proof⟩

```

```

lemma evals_type_sound: "!!E s s'.
  [| wf_java_prog G; G ⊢ (x, s) -es[>] vs -> (x', s'); (x, s) :: ≤E; E ⊢ es[:, :] Ts; G = prg E |]
  ==> (x', s') :: ≤E ∧ (x' = None --> (list_all2 (λv T. G, heap s' ⊢ v :: ≤ T) vs Ts)) ∧ heap s
  ≤ | heap s'"
⟨proof⟩

```

```

lemma exec_type_sound: "!!E s s'.
  [| wf_java_prog G; G ⊢ (x, s) -s0-> (x', s'); (x, s) :: ≤E; E ⊢ s0 √; G = prg E |]
  ==> (x', s') :: ≤E ∧ heap s ≤ | heap s'"
⟨proof⟩

```

```

theorem all_methods_understood:
  "[| G = prg E; wf_java_prog G; G ⊢ (x, s) -e>a'-> Norm s'; a' ≠ Null;
    (x, s) :: ≤E; E ⊢ e :: Class C; method (G, C) sig ≠ None |] ==>
    method (G, fst (the (heap s' (the_Addr a')))) sig ≠ None"
⟨proof⟩

```

```

declare split_beta [simp del]
declare fun_upd_apply [simp]
declare wf_prog_ws_prog [simp del]

```

end

## 2.14 Example MicroJava Program

```
theory Example imports SystemClasses Eval begin
```

The following example MicroJava program includes: class declarations with inheritance, hiding of fields, and overriding of methods (with refined result type), instance creation, local assignment, sequential composition, method call with dynamic binding, literal values, expression statement, local access, type cast, field assignment (in part), skip.

```

class Base {
  boolean vee;
  Base foo(Base x) {return x;}

```

```

}

class Ext extends Base {
  int vee;
  Ext foo(Base x) {((Ext)x).vee=1; return null;}
}

class Example {
  public static void main (String args[]) {
    Base e=new Ext();
    e.foo(null);
  }
}

datatype cnam' = Base' | Ext'
datatype vnam' = vee' | x' | e'

cnam' and vnam' are intended to be isomorphic to cnam and vnam

axiomatization cnam' :: "cnam' => cname"
where
  inj_cnam': "(cnam' x = cnam' y) = (x = y)" and
  surj_cnam': "∃ m. n = cnam' m"

axiomatization vnam' :: "vnam' => vname"
where
  inj_vnam': "(vnam' x = vnam' y) = (x = y)" and
  surj_vnam': "∃ m. n = vnam' m"

declare inj_cnam' [simp] inj_vnam' [simp]

abbreviation Base :: cname
  where "Base == cnam' Base'"
abbreviation Ext :: cname
  where "Ext == cnam' Ext'"
abbreviation vee :: vname
  where "vee == VName (vnam' vee')"
abbreviation x :: vname
  where "x == VName (vnam' x')"
abbreviation e :: vname
  where "e == VName (vnam' e')"

axiomatization where
  Base_not_Object: "Base ≠ Object" and
  Ext_not_Object: "Ext ≠ Object" and
  Base_not_Xcpt: "Base ≠ Xcpt z" and
  Ext_not_Xcpt: "Ext ≠ Xcpt z" and
  e_not_This: "e ≠ This"

declare Base_not_Object [simp] Ext_not_Object [simp]
declare Base_not_Xcpt [simp] Ext_not_Xcpt [simp]
declare e_not_This [simp]
declare Base_not_Object [symmetric, simp]

```

```

declare Ext_not_Object [symmetric, simp]
declare Base_not_Xcpt [symmetric, simp]
declare Ext_not_Xcpt [symmetric, simp]

definition foo_Base :: java_mb
  where "foo_Base == ([x],[],Skip,LAcc x)"

definition foo_Ext :: java_mb
  where "foo_Ext == ([x],[],Expr( {Ext}Cast Ext
    (LAcc x)..vee:=Lit (Intg Numeral1)),
    Lit Null)"

consts foo :: mname

definition BaseC :: "java_mb cdecl"
  where "BaseC == (Base, (Object,
    [(vee, PrimT Boolean)],
    [(foo,[Class Base]),Class Base,foo_Base]))"

definition ExtC :: "java_mb cdecl"
  where "ExtC == (Ext, (Base ,
    [(vee, PrimT Integer)],
    [(foo,[Class Base]),Class Ext,foo_Ext]))"

definition test :: stmt
  where "test == Expr(e::=NewC Ext);;
    Expr({Base}LAcc e..foo({[Class Base]}[Lit Null]))"

consts
  a :: loc
  b :: loc

abbreviation
  NP :: xcpt where
  "NP == NullPointer"

abbreviation
  tprg :: "java_mb prog" where
  "tprg == [ObjectC, BaseC, ExtC, ClassCastC, NullPointerC, OutOfMemoryC]"

abbreviation
  obj1 :: obj where
  "obj1 == (Ext, Map.empty((vee, Base)→Bool False, (vee, Ext )→Intg 0))"

abbreviation "s0 == Norm (Map.empty, Map.empty)"
abbreviation "s1 == Norm (Map.empty(a→obj1),Map.empty(e→Addr a))"
abbreviation "s2 == Norm (Map.empty(a→obj1),Map.empty(x→Null, This→Addr a))"
abbreviation "s3 == (Some NP, Map.empty(a→obj1),Map.empty(e→Addr a))"

lemmas map_of_Cons = map_of.simps(2)

lemma map_of_Cons1 [simp]: "map_of ((aa,bb)#ps) aa = Some bb"
  <proof>
lemma map_of_Cons2 [simp]: "aa≠k ==> map_of ((k,bb)#ps) aa = map_of ps aa"

```

```

⟨proof⟩
declare map_of_Cons [simp del] — sic!

lemma class_tprg_Object [simp]: "class tprg Object = Some (undefined, [], [])"
⟨proof⟩

lemma class_tprg_NP [simp]: "class tprg (Xcpt NP) = Some (Object, [], [])"
⟨proof⟩

lemma class_tprg_OM [simp]: "class tprg (Xcpt OutOfMemory) = Some (Object, [], [])"
⟨proof⟩

lemma class_tprg_CC [simp]: "class tprg (Xcpt ClassCast) = Some (Object, [], [])"
⟨proof⟩

lemma class_tprg_Base [simp]:
  "class tprg Base = Some (Object,
    [(vee, PrimT Boolean)],
    [((foo, [Class Base]), Class Base, foo_Base)])"
⟨proof⟩

lemma class_tprg_Ext [simp]:
  "class tprg Ext = Some (Base,
    [(vee, PrimT Integer)],
    [((foo, [Class Base]), Class Ext, foo_Ext)])"
⟨proof⟩

lemma not_Object_subcls [elim!]: "(Object, C) ∈ (subcls1 tprg)+ ==> R"
⟨proof⟩

lemma subcls_ObjectD [dest!]: "tprg ⊢ Object ≤C C ==> C = Object"
⟨proof⟩

lemma not_Base_subcls_Ext [elim!]: "(Base, Ext) ∈ (subcls1 tprg)+ ==> R"
⟨proof⟩

lemma class_tprgD:
  "class tprg C = Some z ==> C=Object ∨ C=Base ∨ C=Ext ∨ C=Xcpt NP ∨ C=Xcpt ClassCast
  ∨ C=Xcpt OutOfMemory"
⟨proof⟩

lemma not_class_subcls_class [elim!]: "(C, C) ∈ (subcls1 tprg)+ ==> R"
⟨proof⟩

lemma unique_classes: "unique tprg"
⟨proof⟩

lemmas subcls_direct = subcls1I [THEN r_into_rtrancl [where r="subcls1 G"]] for G

lemma Ext_subcls_Base [simp]: "tprg ⊢ Ext ≤C Base"
⟨proof⟩

lemma Ext_widen_Base [simp]: "tprg ⊢ Class Ext ≤ Class Base"
⟨proof⟩

```

```

declare ty_expr_ty_exprs_wt_stmt.intros [intro!]

lemma acyclic_subcls1': "acyclic (subcls1 tprg)"
⟨proof⟩

lemmas wf_subcls1' = acyclic_subcls1' [THEN finite_subcls1 [THEN finite_acyclic_wf_converse]]

lemmas fields_rec' = wf_subcls1' [THEN [2] fields_rec_lemma]

lemma fields_Object [simp]: "fields (tprg, Object) = []"
⟨proof⟩

declare is_class_def [simp]

lemma fields_Base [simp]: "fields (tprg, Base) = [((vee, Base), PrimT Boolean)]"
⟨proof⟩

lemma fields_Ext [simp]:
  "fields (tprg, Ext) = [((vee, Ext ), PrimT Integer)] @ fields (tprg, Base)"
⟨proof⟩

lemmas method_rec' = wf_subcls1' [THEN [2] method_rec_lemma]

lemma method_Object [simp]: "method (tprg, Object) = map_of []"
⟨proof⟩

lemma method_Base [simp]: "method (tprg, Base) = map_of
  [((foo, [Class Base]), Base, (Class Base, foo_Base))]"
⟨proof⟩

lemma method_Ext [simp]: "method (tprg, Ext) = (method (tprg, Base) ++ map_of
  [((foo, [Class Base]), Ext , (Class Ext, foo_Ext))])"
⟨proof⟩

lemma wf_foo_Base:
  "wf_mdecl wf_java_mdecl tprg Base ((foo, [Class Base]), (Class Base, foo_Base))"
⟨proof⟩

lemma wf_foo_Ext:
  "wf_mdecl wf_java_mdecl tprg Ext ((foo, [Class Base]), (Class Ext, foo_Ext))"
⟨proof⟩

lemma wf_ObjectC:
  "ws_cdecl tprg ObjectC ∧
   wf_cdecl_mdecl wf_java_mdecl tprg ObjectC ∧ wf_mrT tprg ObjectC"
⟨proof⟩

lemma wf_NP:
  "ws_cdecl tprg NullPointerC ∧
   wf_cdecl_mdecl wf_java_mdecl tprg NullPointerC ∧ wf_mrT tprg NullPointerC"
⟨proof⟩

lemma wf_OM:

```

```
"ws_cdecl tprg OutOfMemoryC ∧
  wf_cdecl_mdecl wf_java_mdecl tprg OutOfMemoryC ∧ wf_mrT tprg OutOfMemoryC"
⟨proof⟩
```

```
lemma wf_CC:
  "ws_cdecl tprg ClassCastC ∧
    wf_cdecl_mdecl wf_java_mdecl tprg ClassCastC ∧ wf_mrT tprg ClassCastC"
⟨proof⟩
```

```
lemma wf_BaseC:
  "ws_cdecl tprg BaseC ∧
    wf_cdecl_mdecl wf_java_mdecl tprg BaseC ∧ wf_mrT tprg BaseC"
⟨proof⟩
```

```
lemma wf_ExtC:
  "ws_cdecl tprg ExtC ∧
    wf_cdecl_mdecl wf_java_mdecl tprg ExtC ∧ wf_mrT tprg ExtC"
⟨proof⟩
```

```
lemma [simp]: "fst ObjectC = Object" ⟨proof⟩
```

```
lemma wf_tprg:
  "wf_prog wf_java_mdecl tprg"
⟨proof⟩
```

```
lemma appl_methds_foo_Base:
  "appl_methds tprg Base (foo, [NT]) =
    {((Class Base, Class Base), [Class Base])}"
⟨proof⟩
```

```
lemma max_spec_foo_Base: "max_spec tprg Base (foo, [NT]) =
  {((Class Base, Class Base), [Class Base])}"
⟨proof⟩
```

```
lemmas t = ty_expr_ty_exprs_wt_stmt.intros
schematic_goal wt_test: "(tprg, Map.empty(e ↦ Class Base)) ⊢
  Expr(e ::= NewC Ext);; Expr({Base}LAcc e..foo({pTs'}[Lit Null])) ✓"
⟨proof⟩
```

```
lemmas e = NewCI eval_evals_exec.intros
declare if_split [split del]
declare init_vars_def [simp] c_hupd_def [simp] cast_ok_def [simp]
schematic_goal exec_test:
  " [!new_Addr (heap (snd s0)) = (a, None)] ==>
    tprg ⊢ s0 -test-> ?s"
⟨proof⟩
```

```
end
```

## 2.15 Example for generating executable code from Java semantics

```

theory JListExample
imports Eval
begin

declare [[syntax_ambiguity_warning = false]]

consts
  list_nam :: cname
  append_name :: mname

axiomatization val_nam next_nam l_nam l1_nam l2_nam l3_nam l4_nam :: vnam
where distinct_fields: "val_nam  $\neq$  next_nam"
  and distinct_vars1: "l_nam  $\neq$  l1_nam"
  and distinct_vars2: "l_nam  $\neq$  l2_nam"
  and distinct_vars3: "l_nam  $\neq$  l3_nam"
  and distinct_vars4: "l_nam  $\neq$  l4_nam"
  and distinct_vars5: "l1_nam  $\neq$  l2_nam"
  and distinct_vars6: "l1_nam  $\neq$  l3_nam"
  and distinct_vars7: "l1_nam  $\neq$  l4_nam"
  and distinct_vars8: "l2_nam  $\neq$  l3_nam"
  and distinct_vars9: "l2_nam  $\neq$  l4_nam"
  and distinct_vars10: "l3_nam  $\neq$  l4_nam"

lemmas distinct_vars =
  distinct_vars1
  distinct_vars2
  distinct_vars3
  distinct_vars4
  distinct_vars5
  distinct_vars6
  distinct_vars7
  distinct_vars8
  distinct_vars9
  distinct_vars10

definition list_name :: cname where
  "list_name = Cname list_nam"

definition val_name :: vname where
  "val_name == VName val_nam"

definition next_name :: vname where
  "next_name == VName next_nam"

definition l_name :: vname where
  "l_name == VName l_nam"

definition l1_name :: vname where
  "l1_name == VName l1_nam"

definition l2_name :: vname where

```

```
"l2_name == VName l2_nam"
```

```
definition l3_name :: vname where
```

```
"l3_name == VName l3_nam"
```

```
definition l4_name :: vname where
```

```
"l4_name == VName l4_nam"
```

```
definition list_class :: "java_mb class" where
```

```
"list_class ==
  (Object,
   [(val_name, PrimT Integer), (next_name, RefT (ClassT list_name))],
   [(append_name, [RefT (ClassT list_name)]), PrimT Void,
    ([l_name], [],
     If(BinOp Eq ({list_name}(LAcc This)..next_name) (Lit Null))
       Expr ({list_name}(LAcc This)..next_name:=LAcc l_name)
     Else
       Expr ({list_name}({list_name}(LAcc This)..next_name)..
        append_name([RefT (ClassT list_name)] [LAcc l_name])),
    Lit Unit)))"
```

```
definition example_prg :: "java_mb prog" where
```

```
"example_prg == [ObjectC, (list_name, list_class)]"
```

```
code_datatype list_nam
```

```
lemma equal_cnam_code [code]:
```

```
"HOL.equal list_nam list_nam  $\longleftrightarrow$  True"
```

```
<proof>
```

```
code_datatype append_name
```

```
lemma equal_mname_code [code]:
```

```
"HOL.equal append_name append_name  $\longleftrightarrow$  True"
```

```
<proof>
```

```
code_datatype val_nam next_nam l_nam l1_nam l2_nam l3_nam l4_nam
```

```
lemma equal_vnam_code [code]:
```

```
"HOL.equal val_nam val_nam  $\longleftrightarrow$  True"
```

```
"HOL.equal next_nam next_nam  $\longleftrightarrow$  True"
```

```
"HOL.equal l_nam l_nam  $\longleftrightarrow$  True"
```

```
"HOL.equal l1_nam l1_nam  $\longleftrightarrow$  True"
```

```
"HOL.equal l2_nam l2_nam  $\longleftrightarrow$  True"
```

```
"HOL.equal l3_nam l3_nam  $\longleftrightarrow$  True"
```

```
"HOL.equal l4_nam l4_nam  $\longleftrightarrow$  True"
```

```
"HOL.equal val_nam next_nam  $\longleftrightarrow$  False"
```

```
"HOL.equal next_nam val_nam  $\longleftrightarrow$  False"
```

```
"HOL.equal l_nam l1_nam  $\longleftrightarrow$  False"
```

```
"HOL.equal l_nam l2_nam  $\longleftrightarrow$  False"
```

```
"HOL.equal l_nam l3_nam  $\longleftrightarrow$  False"
```

```
"HOL.equal l_nam l4_nam  $\longleftrightarrow$  False"
```

```
"HOL.equal l1_nam l_nam  $\longleftrightarrow$  False"
```

```
"HOL.equal l1_nam l2_nam  $\longleftrightarrow$  False"
```



```

"HOL.equal l1_name l3_name  $\longleftrightarrow$  False"
"HOL.equal l1_name l4_name  $\longleftrightarrow$  False"

"HOL.equal l2_name l1_name  $\longleftrightarrow$  False"
"HOL.equal l2_name l1_name  $\longleftrightarrow$  False"
"HOL.equal l2_name l3_name  $\longleftrightarrow$  False"
"HOL.equal l2_name l4_name  $\longleftrightarrow$  False"

"HOL.equal l3_name l1_name  $\longleftrightarrow$  False"
"HOL.equal l3_name l1_name  $\longleftrightarrow$  False"
"HOL.equal l3_name l2_name  $\longleftrightarrow$  False"
"HOL.equal l3_name l4_name  $\longleftrightarrow$  False"

"HOL.equal l4_name l1_name  $\longleftrightarrow$  False"
"HOL.equal l4_name l1_name  $\longleftrightarrow$  False"
"HOL.equal l4_name l2_name  $\longleftrightarrow$  False"
"HOL.equal l4_name l3_name  $\longleftrightarrow$  False"
<proof>

```

axiomatization where

```
nat_to_loc'_inject: "nat_to_loc' l = nat_to_loc' l'  $\longleftrightarrow$  l = l'"
```

lemma equal\_loc'\_code [code]:

```

"HOL.equal (nat_to_loc' l) (nat_to_loc' l')  $\longleftrightarrow$  l = l'"
<proof>

```

definition undefined\_cname :: cname

```
where [code del]: "undefined_cname = undefined"
```

declare undefined\_cname\_def[symmetric, code\_unfold]

code\_datatype Object Xcpt Cname undefined\_cname

definition undefined\_val :: val

```
where [code del]: "undefined_val = undefined"
```

declare undefined\_val\_def[symmetric, code\_unfold]

code\_datatype Unit Null Bool Intg Addr undefined\_val

definition E where

```

"E = Expr (l1_name::=NewC list_name);;
  Expr ({list_name}(LAcc l1_name)..val_name:=Lit (Intg 1));;
  Expr (l2_name::=NewC list_name);;
  Expr ({list_name}(LAcc l2_name)..val_name:=Lit (Intg 2));;
  Expr (l3_name::=NewC list_name);;
  Expr ({list_name}(LAcc l3_name)..val_name:=Lit (Intg 3));;
  Expr (l4_name::=NewC list_name);;
  Expr ({list_name}(LAcc l4_name)..val_name:=Lit (Intg 4));;
  Expr ({list_name}(LAcc l1_name)..
    append_name({[RefT (ClassT list_name)]}[LAcc l2_name]));;
  Expr ({list_name}(LAcc l1_name)..
    append_name({[RefT (ClassT list_name)]}[LAcc l3_name]));;
  Expr ({list_name}(LAcc l1_name)..
    append_name({[RefT (ClassT list_name)]}[LAcc l4_name]))"
```

definition test where

```
"test = Predicate.Pred ( $\lambda$ s. example_prg $\vdash$ Norm (Map.empty, Map.empty)  $\neg$ E $\rightarrow$  s)"
```

```
lemma test_code [code]:  
  "test = exec_i_i_i_o example_prg (Norm (Map.empty, Map.empty)) E"  
  ⟨proof⟩  
  
  ⟨ML⟩  
  
end
```

# Chapter 3

## Java Virtual Machine

### 3.1 State of the JVM

```
theory JVMState
imports "../J/Conform"
begin
```

#### 3.1.1 Frame Stack

```
type_synonym opstack = "val list"
type_synonym locvars = "val list"
type_synonym p_count = nat
```

```
type_synonym
  frame = "opstack ×
           locvars ×
           cname ×
           sig ×
           p_count"
```

- operand stack
- local variables (including this pointer and method parameters)
- name of class where current method is defined
- method name + parameter types
- program counter within frame

#### 3.1.2 Exceptions

```
definition raise_system_xcpt :: "bool ⇒ xcpt ⇒ val option" where
  "raise_system_xcpt b x ≡ raise_if b x None"
```

#### 3.1.3 Runtime State

```
type_synonym
  jvm_state = "val option × aheap × frame list" — exception flag, heap, frames
```

#### 3.1.4 Lemmas

```
lemma new_Addr_OutOfMemory:
  "snd (new_Addr hp) = Some xcp ⇒ xcp = Addr (XcptRef OutOfMemory)"
⟨proof⟩
```

end

## 3.2 Instructions of the JVM

theory *JVMInstructions* imports *JVMState* begin

**datatype**

```

  instr = Load nat           — load from local variable
        | Store nat          — store into local variable
        | LitPush val        — push a literal (constant)
        | New cname          — create object
        | Getfield vname cname — Fetch field from object
        | Putfield vname cname — Set field in object
        | Checkcast cname    — Check whether object is of given type
        | Invoke cname mname "(ty list)" — inv. instance meth of an object
        | Return             — return from method
        | Pop                 — pop top element from opstack
        | Dup                 — duplicate top element of opstack
        | Dup_x1              — duplicate top element and push 2 down
        | Dup_x2              — duplicate top element and push 3 down
        | Swap                — swap top and next to top element
        | IAdd                — integer addition
        | Goto int            — goto relative address
        | Ifcmpeq int         — branch if int/ref comparison succeeds
        | Throw               — throw top of stack as exception

```

**type\_synonym**

```

  bytecode = "instr list"

```

**type\_synonym**

```

  exception_entry = "p_count × p_count × p_count × cname"
                  — start-pc, end-pc, handler-pc, exception type

```

**type\_synonym**

```

  exception_table = "exception_entry list"

```

**type\_synonym**

```

  jvm_method = "nat × nat × bytecode × exception_table"
              — max stacksize, size of register set, instruction sequence, handler table

```

**type\_synonym**

```

  jvm_prog = "jvm_method prog"

```

end

## 3.3 JVM Instruction Semantics

theory *JVMExecInstr* imports *JVMInstructions* *JVMState* begin

```

primrec exec_instr :: "[instr, jvm_prog, aheap, opstack, locvars, cname, sig, p_count,
frame list] => jvm_state"

```

where

```

  "exec_instr (Load idx) G hp stk vars Cl sig pc frs =
    (None, hp, ((vars ! idx) # stk, vars, Cl, sig, pc+1)#frs)" |

```

```

"exec_instr (Store idx) G hp stk vars Cl sig pc frs =
  (None, hp, (tl stk, vars[idx:=hd stk], Cl, sig, pc+1)#frs)" |

"exec_instr (LitPush v) G hp stk vars Cl sig pc frs =
  (None, hp, (v # stk, vars, Cl, sig, pc+1)#frs)" |

"exec_instr (New C) G hp stk vars Cl sig pc frs =
  (let (oref,xp') = new_Addr hp;
       fs   = init_vars (fields(G,C));
       hp'  = if xp'=None then hp(oref ↦ (C,fs)) else hp;
       pc'  = if xp'=None then pc+1 else pc
   in
    (xp', hp', (Addr oref#stk, vars, Cl, sig, pc')#frs))" |

"exec_instr (Getfield F C) G hp stk vars Cl sig pc frs =
  (let oref = hd stk;
       xp'  = raise_system_xcpt (oref=None) NullPointer;
       (oc,fs) = the(hp(the_Addr oref));
       pc'  = if xp'=None then pc+1 else pc
   in
    (xp', hp, (the(fs(F,C))#(tl stk), vars, Cl, sig, pc')#frs))" |

"exec_instr (Putfield F C) G hp stk vars Cl sig pc frs =
  (let (fval,oref)= (hd stk, hd(tl stk));
       xp'  = raise_system_xcpt (oref=None) NullPointer;
       a    = the_Addr oref;
       (oc,fs) = the(hp a);
       hp'  = if xp'=None then hp(a ↦ (oc, fs((F,C) ↦ fval))) else hp;
       pc'  = if xp'=None then pc+1 else pc
   in
    (xp', hp', (tl (tl stk), vars, Cl, sig, pc')#frs))" |

"exec_instr (Checkcast C) G hp stk vars Cl sig pc frs =
  (let oref = hd stk;
       xp'  = raise_system_xcpt (¬ cast_ok G C hp oref) ClassCast;
       stk' = if xp'=None then stk else tl stk;
       pc'  = if xp'=None then pc+1 else pc
   in
    (xp', hp, (stk', vars, Cl, sig, pc')#frs))" |

"exec_instr (Invoke C mn ps) G hp stk vars Cl sig pc frs =
  (let n      = length ps;
       argsoref = take (n+1) stk;
       oref = last argsoref;
       xp'  = raise_system_xcpt (oref=None) NullPointer;
       dynT = fst(the(hp(the_Addr oref)));
       (dc,mh,mxs,mxl,c)= the (method (G,dynT) (mn,ps));
       frs' = if xp'=None then
                [([],rev argsoref@replicate mxl undefined,dc,(mn,ps),0)]
              else []
   in
    (xp', hp, frs'@(stk, vars, Cl, sig, pc)#frs))" |

```

— Because exception handling needs the pc of the Invoke instruction,

— Invoke doesn't change stk and pc yet (Return does that).

```
"exec_instr Return G hp stk0 vars Cl sig0 pc frs =
  (if frs=[] then
    (None, hp, [])
  else
    let val = hd stk0; (stk,loc,C,sig,pc) = hd frs;
      (mn,pt) = sig0; n = length pt
    in
      (None, hp, (val#(drop (n+1) stk),loc,C,sig,pc+1)#tl frs))"
```

— Return drops arguments from the caller's stack and increases

— the program counter in the caller /

```
"exec_instr Pop G hp stk vars Cl sig pc frs =
  (None, hp, (tl stk, vars, Cl, sig, pc+1)#frs)" /
```

```
"exec_instr Dup G hp stk vars Cl sig pc frs =
  (None, hp, (hd stk # stk, vars, Cl, sig, pc+1)#frs)" /
```

```
"exec_instr Dup_x1 G hp stk vars Cl sig pc frs =
  (None, hp, (hd stk # hd (tl stk) # hd stk # (tl (tl stk)),
    vars, Cl, sig, pc+1)#frs)" /
```

```
"exec_instr Dup_x2 G hp stk vars Cl sig pc frs =
  (None, hp,
    (hd stk # hd (tl stk) # (hd (tl (tl stk))) # hd stk # (tl (tl (tl stk)))),
    vars, Cl, sig, pc+1)#frs)" /
```

```
"exec_instr Swap G hp stk vars Cl sig pc frs =
  (let (val1,val2) = (hd stk,hd (tl stk))
  in
    (None, hp, (val2#val1#(tl (tl stk)), vars, Cl, sig, pc+1)#frs))" /
```

```
"exec_instr IAdd G hp stk vars Cl sig pc frs =
  (let (val1,val2) = (hd stk,hd (tl stk))
  in
    (None, hp, (Intg ((the_Intg val1)+(the_Intg val2))#(tl (tl stk)),
      vars, Cl, sig, pc+1)#frs))" /
```

```
"exec_instr (Ifcmpeq i) G hp stk vars Cl sig pc frs =
  (let (val1,val2) = (hd stk, hd (tl stk));
    pc' = if val1 = val2 then nat(int pc+i) else pc+1
  in
    (None, hp, (tl (tl stk), vars, Cl, sig, pc')#frs))" /
```

```
"exec_instr (Goto i) G hp stk vars Cl sig pc frs =
  (None, hp, (stk, vars, Cl, sig, nat(int pc+i))#frs)" /
```

```
"exec_instr Throw G hp stk vars Cl sig pc frs =
  (let xcpt = raise_system_xcpt (hd stk = Null) NullPointer;
    xcpt' = if xcpt = None then Some (hd stk) else xcpt
  in
    (xcpt', hp, (stk, vars, Cl, sig, pc)#frs))"
```

end

### 3.4 Exception handling in the JVM

theory JVMExceptions imports JVMInstructions begin

definition match\_exception\_entry :: "jvm\_prog  $\Rightarrow$  cname  $\Rightarrow$  p\_count  $\Rightarrow$  exception\_entry  $\Rightarrow$  bool" where

"match\_exception\_entry G cn pc ee ==  
 let (start\_pc, end\_pc, handler\_pc, catch\_type) = ee in  
 start\_pc  $\leq$  pc  $\wedge$  pc < end\_pc  $\wedge$  G  $\vdash$  cn  $\preceq_C$  catch\_type"

primrec match\_exception\_table :: "jvm\_prog  $\Rightarrow$  cname  $\Rightarrow$  p\_count  $\Rightarrow$  exception\_table  
 $\Rightarrow$  p\_count option"

where

"match\_exception\_table G cn pc [] = None"  
 | "match\_exception\_table G cn pc (e#es) = (if match\_exception\_entry G cn pc e  
 then Some (fst (snd (snd e)))  
 else match\_exception\_table G cn pc es)"

abbreviation

ex\_table\_of :: "jvm\_method  $\Rightarrow$  exception\_table"  
 where "ex\_table\_of m == snd (snd (snd m))"

primrec find\_handler :: "jvm\_prog  $\Rightarrow$  val option  $\Rightarrow$  aheap  $\Rightarrow$  frame list  
 $\Rightarrow$  jvm\_state"

where

"find\_handler G xcpt hp [] = (xcpt, hp, [])"  
 | "find\_handler G xcpt hp (fr#frs) =  
 (case xcpt of  
 None  $\Rightarrow$  (None, hp, fr#frs)  
 | Some xc  $\Rightarrow$   
 let (stk, loc, C, sig, pc) = fr in  
 (case match\_exception\_table G (cname\_of hp xc) pc  
 (ex\_table\_of (snd(snd(the(method (G, C) sig))))) of  
 None  $\Rightarrow$  find\_handler G (Some xc) hp frs  
 | Some handler\_pc  $\Rightarrow$  (None, hp, ([xc], loc, C, sig, handler\_pc)#frs)))"

System exceptions are allocated in all heaps:

Only program counters that are mentioned in the exception table can be returned by *match\_exception\_table*:

lemma match\_exception\_table\_in\_et:

"match\_exception\_table G C pc et = Some pc'  $\implies \exists e \in \text{set et. pc}' = \text{fst (snd (snd e))}"$   
 <proof>

end

### 3.5 Program Execution in the JVM

theory JVMEExec imports JVMEExecInstr JVMExceptions begin

```

fun
  exec :: "jvm_prog × jvm_state => jvm_state option"
  — exec is not recursive. fun is just used for pattern matching
where
  "exec (G, xp, hp, []) = None"

  | "exec (G, None, hp, (stk,loc,C,sig,pc)#frs) =
    (let
      i = fst(snd(snd(snd(snd(the(method (G,C) sig))))) ! pc;
      (xcpt', hp', frs') = exec_instr i G hp stk loc C sig pc frs
    in Some (find_handler G xcpt' hp' frs'))"

  | "exec (G, Some xp, hp, frs) = None"

```

```

definition exec_all :: "[jvm_prog,jvm_state,jvm_state] => bool"
  (<_ ⊢ _ -jvm→ _> [61,61,61]60) where
  "G ⊢ s -jvm→ t == (s,t) ∈ {(s,t). exec(G,s) = Some t}"

```

The start configuration of the JVM: in the start heap, we call a method  $m$  of class  $C$  in program  $G$ . The *this* pointer of the frame is set to *Null* to simulate a static method invocation.

```

definition start_state :: "jvm_prog ⇒ cname ⇒ mname ⇒ jvm_state" where
  "start_state G C m ≡
  let (C',rT,mxs,mxl,i,et) = the (method (G,C) (m,[])) in
  (None, start_heap G, [([], Null # replicate mxl undefined, C, (m,[]), 0)])"

```

**end**

### 3.6 Example for generating executable code from JVM semantics

```

theory JVMListExample
imports "../J/SystemClasses" JVExec
begin

```

Since the types *cnam*, *vnam*, and *mname* are anonymous, we describe distinctness of names in the example by axioms:

```

axiomatization list_nam test_nam :: cname
  where distinct_classes: "list_nam ≠ test_nam"

```

```

axiomatization append_name makelist_name :: mname
  where distinct_methods: "append_name ≠ makelist_name"

```

```

axiomatization val_nam next_nam :: vnam
  where distinct_fields: "val_nam ≠ next_nam"

```

```

axiomatization
  where nat_to_loc'_inject: "nat_to_loc' l = nat_to_loc' l' ⟷ l = l'"

```

```

definition list_name :: cname

```



```

where "list_name = Cname list_nam"

definition test_name :: cname
  where "test_name = Cname test_nam"

definition val_name :: vname
  where "val_name = VName val_nam"

definition next_name :: vname
  where "next_name = VName next_nam"

definition append_ins :: bytecode where
  "append_ins =
    [Load 0,
     Getfield next_name list_name,
     Dup,
     LitPush Null,
     Ifcmpeq 4,
     Load 1,
     Invoke list_name append_name [Class list_name],
     Return,
     Pop,
     Load 0,
     Load 1,
     Putfield next_name list_name,
     LitPush Unit,
     Return]"

definition list_class :: "jvm_method class" where
  "list_class =
    (Object,
     [(val_name, PrimT Integer), (next_name, Class list_name)],
     [(append_name, [Class list_name]), PrimT Void,
      (3, 0, append_ins, [(1,2,8,Xcpt NullPointer)]))])"

definition make_list_ins :: bytecode where
  "make_list_ins =
    [New list_name,
     Dup,
     Store 0,
     LitPush (Intg 1),
     Putfield val_name list_name,
     New list_name,
     Dup,
     Store 1,
     LitPush (Intg 2),
     Putfield val_name list_name,
     New list_name,
     Dup,
     Store 2,
     LitPush (Intg 3),
     Putfield val_name list_name,
     Load 0,
     Load 1,

```

```

    Invoke list_name append_name [Class list_name],
    Pop,
    Load 0,
    Load 2,
    Invoke list_name append_name [Class list_name],
    Return]"

```

**definition** test\_class :: "jvm\_method class" where

```

"test_class =
  (Object, [],
   [(makelist_name, []), PrimT Void, (3, 2, make_list_ins, [])])]"

```

**definition** E :: jvm\_prog where

```

"E = SystemClasses @ [(list_name, list_class), (test_name, test_class)]"
```

**code\_datatype** list\_nam test\_nam

**lemma** equal\_cnam\_code [code]:

```

"HOL.equal list_nam list_nam  $\longleftrightarrow$  True"
"HOL.equal test_nam test_nam  $\longleftrightarrow$  True"
"HOL.equal list_nam test_nam  $\longleftrightarrow$  False"
"HOL.equal test_nam list_nam  $\longleftrightarrow$  False"
<proof>

```

**code\_datatype** append\_name makelist\_name

**lemma** equal\_mname\_code [code]:

```

"HOL.equal append_name append_name  $\longleftrightarrow$  True"
"HOL.equal makelist_name makelist_name  $\longleftrightarrow$  True"
"HOL.equal append_name makelist_name  $\longleftrightarrow$  False"
"HOL.equal makelist_name append_name  $\longleftrightarrow$  False"
<proof>

```

**code\_datatype** val\_nam next\_nam

**lemma** equal\_vnam\_code [code]:

```

"HOL.equal val_nam val_nam  $\longleftrightarrow$  True"
"HOL.equal next_nam next_nam  $\longleftrightarrow$  True"
"HOL.equal val_nam next_nam  $\longleftrightarrow$  False"
"HOL.equal next_nam val_nam  $\longleftrightarrow$  False"
<proof>

```

**lemma** equal\_loc'\_code [code]:

```

"HOL.equal (nat_to_loc' l) (nat_to_loc' l')  $\longleftrightarrow$  l = l'"
<proof>

```

**definition** undefined\_cname :: cname

```

  where [code del]: "undefined_cname = undefined"

```

**code\_datatype** Object Xcpt Cname undefined\_cname

**declare** undefined\_cname\_def[symmetric, code\_unfold]

**definition** undefined\_val :: val

```

  where [code del]: "undefined_val = undefined"

```

**declare** undefined\_val\_def[symmetric, code\_unfold]

**code\_datatype** Unit Null Bool Intg Addr undefined\_val

**definition**

```
"test = exec (E, start_state E test_name makelist_name)"
```

$\langle ML \rangle$

**end**

### 3.7 A Defensive JVM

**theory** JVMDefensive

**imports** JVExec

**begin**

Extend the state space by one element indicating a type error (or other abnormal termination)

```
datatype 'a type_error = TypeError | Normal 'a
```

**abbreviation**

```
fifth :: "'a × 'b × 'c × 'd × 'e × 'f ⇒ 'e"
where "fifth x == fst(snd(snd(snd(snd x))))"
```

```
fun isAddr :: "val ⇒ bool" where
```

```
  "isAddr (Addr loc) = True"
```

```
| "isAddr v          = False"
```

```
fun isIntg :: "val ⇒ bool" where
```

```
  "isIntg (Intg i) = True"
```

```
| "isIntg v          = False"
```

```
definition isRef :: "val ⇒ bool" where
```

```
  "isRef v ≡ v = Null ∨ isAddr v"
```

```
primrec check_instr :: "[instr, jvm_prog, aheap, opstack, locvars,  
                        cname, sig, p_count, nat, frame list] ⇒ bool" where
```

```
  "check_instr (Load idx) G hp stk vars C sig pc mxs frs =  
    (idx < length vars ∧ size stk < mxs)"
```

```
| "check_instr (Store idx) G hp stk vars Cl sig pc mxs frs =  
    (0 < length stk ∧ idx < length vars)"
```

```
| "check_instr (LitPush v) G hp stk vars Cl sig pc mxs frs =  
    (¬isAddr v ∧ size stk < mxs)"
```

```
| "check_instr (New C) G hp stk vars Cl sig pc mxs frs =  
    (is_class G C ∧ size stk < mxs)"
```

```
| "check_instr (Getfield F C) G hp stk vars Cl sig pc mxs frs =  
    (0 < length stk ∧ is_class G C ∧ field (G,C) F ≠ None ∧  
    (let (C', T) = the (field (G,C) F); ref = hd stk in  
      C' = C ∧ isRef ref ∧ (ref ≠ Null →  
        hp (the_Addr ref) ≠ None ∧  
        (let (D, vs) = the (hp (the_Addr ref)) in  
          G ⊢ D ≤C C ∧ vs (F,C) ≠ None ∧ G, hp ⊢ the (vs (F,C)) :: ≤ T))))"
```

```

| "check_instr (Putfield F C) G hp stk vars Cl sig pc mxs frs =
  (1 < length stk ∧ is_class G C ∧ field (G,C) F ≠ None ∧
   (let (C', T) = the (field (G,C) F); v = hd stk; ref = hd (tl stk) in
    C' = C ∧ isRef ref ∧ (ref ≠ Null →
      hp (the_Addr ref) ≠ None ∧
      (let (D,vs) = the (hp (the_Addr ref)) in
        G ⊢ D ⊆C C ∧ G, hp ⊢ v ::⊆ T))))"

| "check_instr (Checkcast C) G hp stk vars Cl sig pc mxs frs =
  (0 < length stk ∧ is_class G C ∧ isRef (hd stk))"

| "check_instr (Invoke C mn ps) G hp stk vars Cl sig pc mxs frs =
  (length ps < length stk ∧
   (let n = length ps; v = stk!n in
    isRef v ∧ (v ≠ Null →
      hp (the_Addr v) ≠ None ∧
      method (G, cname_of hp v) (mn, ps) ≠ None ∧
      list_all2 (λv T. G, hp ⊢ v ::⊆ T) (rev (take n stk)) ps))))"

| "check_instr Return G hp stk0 vars Cl sig0 pc mxs frs =
  (0 < length stk0 ∧ (0 < length frs →
    method (G, Cl) sig0 ≠ None ∧
    (let v = hd stk0; (C, rT, body) = the (method (G, Cl) sig0) in
      Cl = C ∧ G, hp ⊢ v ::⊆ rT)))"

| "check_instr Pop G hp stk vars Cl sig pc mxs frs =
  (0 < length stk)"

| "check_instr Dup G hp stk vars Cl sig pc mxs frs =
  (0 < length stk ∧ size stk < mxs)"

| "check_instr Dup_x1 G hp stk vars Cl sig pc mxs frs =
  (1 < length stk ∧ size stk < mxs)"

| "check_instr Dup_x2 G hp stk vars Cl sig pc mxs frs =
  (2 < length stk ∧ size stk < mxs)"

| "check_instr Swap G hp stk vars Cl sig pc mxs frs =
  (1 < length stk)"

| "check_instr IAdd G hp stk vars Cl sig pc mxs frs =
  (1 < length stk ∧ isIntg (hd stk) ∧ isIntg (hd (tl stk)))"

| "check_instr (Ifcmpeq b) G hp stk vars Cl sig pc mxs frs =
  (1 < length stk ∧ 0 ≤ int pc+b)"

| "check_instr (Goto b) G hp stk vars Cl sig pc mxs frs =
  (0 ≤ int pc+b)"

| "check_instr Throw G hp stk vars Cl sig pc mxs frs =
  (0 < length stk ∧ isRef (hd stk))"

```

**definition** check :: "jvm\_prog ⇒ jvm\_state ⇒ bool" where

```
"check G s ≡ let (xcpt, hp, frs) = s in
  (case frs of [] ⇒ True | (stk,loc,C,sig,pc)#frs' ⇒
    (let (C',rt,mxs,mxl,ins,et) = the (method (G,C) sig); i = ins!pc in
      pc < size ins ∧
      check_instr i G hp stk loc C sig pc mxs frs'))"
```

**definition** `exec_d` :: "jvm\_prog ⇒ jvm\_state type\_error ⇒ jvm\_state option type\_error"  
**where**

```
"exec_d G s ≡ case s of
  TypeError ⇒ TypeError
| Normal s' ⇒ if check G s' then Normal (exec (G, s')) else TypeError"
```

**definition**

```
exec_all_d :: "jvm_prog ⇒ jvm_state type_error ⇒ jvm_state type_error ⇒ bool"
  (<_ ⊢ _ -jvmd→ _> [61,61,61]60) where
"G ⊢ s -jvmd→ t ⇔
  (s,t) ∈ ({(s,t). exec_d G s = TypeError ∧ t = TypeError} ∪
    {(s,t). ∃t'. exec_d G s = Normal (Some t') ∧ t = Normal t'})"
```

**declare** `split_paired_All` [simp del]

**declare** `split_paired_Ex` [simp del]

**lemma** [dest!]:

```
"(if P then A else B) ≠ B ⇒ P"
⟨proof⟩
```

**lemma** `exec_d_no_errorI` [intro]:

```
"check G s ⇒ exec_d G (Normal s) ≠ TypeError"
⟨proof⟩
```

**theorem** `no_type_error_commutes`:

```
"exec_d G (Normal s) ≠ TypeError ⇒
  exec_d G (Normal s) = Normal (exec (G, s))"
⟨proof⟩
```

**lemma** `defensive_imp_aggressive`:

```
"G ⊢ (Normal s) -jvmd→ (Normal t) ⇒ G ⊢ s -jvm→ t"
⟨proof⟩
```

**end**



## Chapter 4

# Bytecode Verifier

### 4.1 Semilattices

```
theory Semilat
imports Main "HOL-Library.While_Combinator"
begin

type_synonym 'a ord = "'a  $\Rightarrow$  'a  $\Rightarrow$  bool"
type_synonym 'a binop = "'a  $\Rightarrow$  'a  $\Rightarrow$  'a"
type_synonym 'a sl = "'a set  $\times$  'a ord  $\times$  'a binop"

definition lesub :: "'a  $\Rightarrow$  'a ord  $\Rightarrow$  'a  $\Rightarrow$  bool"
  where "lesub x r y  $\longleftrightarrow$  r x y"

definition lesssub :: "'a  $\Rightarrow$  'a ord  $\Rightarrow$  'a  $\Rightarrow$  bool"
  where "lesssub x r y  $\longleftrightarrow$  lesub x r y  $\wedge$  x  $\neq$  y"

definition plussub :: "'a  $\Rightarrow$  ('a  $\Rightarrow$  'b  $\Rightarrow$  'c)  $\Rightarrow$  'b  $\Rightarrow$  'c"
  where "plussub x f y = f x y"

notation (ASCII)
  "lesub"  (<(_ /<= ' _ _)> [50, 1000, 51] 50) and
  "lesssub" (<(_ /< ' _ _)> [50, 1000, 51] 50) and
  "plussub" (<(_ /+ ' _ _)> [65, 1000, 66] 65)

notation
  "lesub"  (<(_ / $\sqsubseteq$  _ _)> [50, 0, 51] 50) and
  "lesssub" (<(_ / $\sqsubset$  _ _)> [50, 0, 51] 50) and
  "plussub" (<(_ /\sqcup _ _)> [65, 0, 66] 65)

abbreviation (input)
  lesub1 :: "'a  $\Rightarrow$  'a ord  $\Rightarrow$  'a  $\Rightarrow$  bool" (<(_ / $\sqsubseteq$  _ _)> [50, 1000, 51] 50)
  where "x  $\sqsubseteq_r$  y == x  $\sqsubseteq_r$  y"

abbreviation (input)
  lesssub1 :: "'a  $\Rightarrow$  'a ord  $\Rightarrow$  'a  $\Rightarrow$  bool" (<(_ / $\sqsubset$  _ _)> [50, 1000, 51] 50)
  where "x  $\sqsubset_r$  y == x  $\sqsubset_r$  y"

abbreviation (input)
```

```

plussub1 :: "'a ⇒ ('a ⇒ 'b ⇒ 'c) ⇒ 'b ⇒ 'c" (<(_ /⊔ _)> [65, 1000, 66] 65)
where "x ⊔f y == x ⊔f y"

definition ord :: "'a × 'a) set ⇒ 'a ord" where
  "ord r ≡ λx y. (x,y) ∈ r"

definition order :: "'a ord ⇒ bool" where
  "order r ≡ (∀x. x ⊆r x) ∧ (∀x y. x ⊆r y ∧ y ⊆r x → x=y) ∧ (∀x y z. x ⊆r y ∧ y ⊆r z → x ⊆r z)"

definition top :: "'a ord ⇒ 'a ⇒ bool" where
  "top r T ≡ ∀x. x ⊆r T"

definition acc :: "'a ord ⇒ bool" where
  "acc r ≡ wf {(y,x). x ⊆r y}"

definition closed :: "'a set ⇒ 'a binop ⇒ bool" where
  "closed A f ≡ ∀x∈A. ∀y∈A. x ⊔f y ∈ A"

definition semilat :: "'a sl ⇒ bool" where
  "semilat ≡ λ(A,r,f). order r ∧ closed A f ∧
    (∀x∈A. ∀y∈A. x ⊆r x ⊔f y) ∧
    (∀x∈A. ∀y∈A. y ⊆r x ⊔f y) ∧
    (∀x∈A. ∀y∈A. ∀z∈A. x ⊆r z ∧ y ⊆r z → x ⊔f y ⊆r z)"

definition is_ub :: "('a × 'a) set ⇒ 'a ⇒ 'a ⇒ 'a ⇒ bool" where
  "is_ub r x y u ≡ (x,u)∈r ∧ (y,u)∈r"

definition is_lub :: "('a × 'a) set ⇒ 'a ⇒ 'a ⇒ 'a ⇒ bool" where
  "is_lub r x y u ≡ is_ub r x y u ∧ (∀z. is_ub r x y z → (u,z)∈r)"

definition some_lub :: "('a × 'a) set ⇒ 'a ⇒ 'a ⇒ 'a" where
  "some_lub r x y ≡ SOME z. is_lub r x y z"

locale Semilat =
  fixes A :: "'a set"
  fixes r :: "'a ord"
  fixes f :: "'a binop"
  assumes semilat: "semilat (A, r, f)"

lemma order_refl [simp, intro]: "order r ⇒ x ⊆r x"
  <proof>

lemma order_antisym: "[| order r; x ⊆r y; y ⊆r x |] ⇒ x = y"
  <proof>

lemma order_trans: "[| order r; x ⊆r y; y ⊆r z |] ⇒ x ⊆r z"
  <proof>

lemma order_less_irrefl [intro, simp]: "order r ⇒ ¬ x ⊆r x"
  <proof>

lemma order_less_trans: "[| order r; x ⊆r y; y ⊆r z |] ⇒ x ⊆r z"
  <proof>

lemma topD [simp, intro]: "top r T ⇒ x ⊆r T"
  <proof>

lemma top_le_conv [simp]: "[| order r; top r T |] ⇒ (T ⊆r x) = (x = T)"
  <proof>

```



**lemma** *semilat\_Def*:

"semilat(A,r,f)  $\equiv$  order r  $\wedge$  closed A f  $\wedge$   
 $(\forall x \in A. \forall y \in A. x \sqsubseteq_r x \sqcup_f y) \wedge$   
 $(\forall x \in A. \forall y \in A. y \sqsubseteq_r x \sqcup_f y) \wedge$   
 $(\forall x \in A. \forall y \in A. \forall z \in A. x \sqsubseteq_r z \wedge y \sqsubseteq_r z \longrightarrow x \sqcup_f y \sqsubseteq_r z)"$

*<proof>*

**lemma** (in Semilat) *orderI* [simp, intro]: "order r"

*<proof>*

**lemma** (in Semilat) *closedI* [simp, intro]: "closed A f"

*<proof>*

**lemma** *closedD*: "[closed A f;  $x \in A$ ;  $y \in A$ ]  $\implies x \sqcup_f y \in A$ "

*<proof>*

**lemma** *closed\_UNIV* [simp]: "closed UNIV f"

*<proof>*

**lemma** (in Semilat) *closed\_f* [simp, intro]: "[ $x \in A$ ;  $y \in A$ ]  $\implies x \sqcup_f y \in A$ "

*<proof>*

**lemma** (in Semilat) *refl\_r* [intro, simp]: " $x \sqsubseteq_r x$ " *<proof>*

**lemma** (in Semilat) *antisym\_r* [intro?]: "[ $x \sqsubseteq_r y$ ;  $y \sqsubseteq_r x$ ]  $\implies x = y$ "

*<proof>*

**lemma** (in Semilat) *trans\_r* [trans, intro?]: "[ $x \sqsubseteq_r y$ ;  $y \sqsubseteq_r z$ ]  $\implies x \sqsubseteq_r z$ "

*<proof>*

**lemma** (in Semilat) *ub1* [simp, intro?]: "[ $x \in A$ ;  $y \in A$ ]  $\implies x \sqsubseteq_r x \sqcup_f y$ "

*<proof>*

**lemma** (in Semilat) *ub2* [simp, intro?]: "[ $x \in A$ ;  $y \in A$ ]  $\implies y \sqsubseteq_r x \sqcup_f y$ "

*<proof>*

**lemma** (in Semilat) *lub* [simp, intro?]:

"[ $x \sqsubseteq_r z$ ;  $y \sqsubseteq_r z$ ;  $x \in A$ ;  $y \in A$ ;  $z \in A$ ]  $\implies x \sqcup_f y \sqsubseteq_r z$ "

*<proof>*

**lemma** (in Semilat) *plus\_le\_conv* [simp]:

"[ $x \in A$ ;  $y \in A$ ;  $z \in A$ ]  $\implies (x \sqcup_f y \sqsubseteq_r z) = (x \sqsubseteq_r z \wedge y \sqsubseteq_r z)"$

*<proof>*

**lemma** (in Semilat) *le\_iff\_plus\_unchanged*: "[ $x \in A$ ;  $y \in A$ ]  $\implies (x \sqsubseteq_r y) = (x \sqcup_f y = y)"$  *<proof>*

**lemma** (in Semilat) *le\_iff\_plus\_unchanged2*: "[ $x \in A$ ;  $y \in A$ ]  $\implies (x \sqsubseteq_r y) = (y \sqcup_f x = y)"$  *<proof>*

**lemma** (in Semilat) *plus\_assoc* [simp]:

assumes a: " $a \in A$ " and b: " $b \in A$ " and c: " $c \in A$ "

shows " $a \sqcup_f (b \sqcup_f c) = a \sqcup_f b \sqcup_f c$ " *<proof>*

**lemma** (in Semilat) *plus\_com\_lemma*:

"[ $a \in A$ ;  $b \in A$ ]  $\implies a \sqcup_f b \sqsubseteq_r b \sqcup_f a$ " *<proof>*

**lemma** (in Semilat) *plus\_commutative*:

"[ $a \in A$ ;  $b \in A$ ]  $\implies a \sqcup_f b = b \sqcup_f a$ "

*<proof>*

**lemma** *is\_lubD*:

"is\_lub r x y u  $\implies$  is\_ub r x y u  $\wedge$  ( $\forall z. \text{is\_ub } r \ x \ y \ z \longrightarrow (u,z) \in r$ )"

*<proof>*

**lemma** *is\_ubI*:

"[ $(x,u) \in r$ ;  $(y,u) \in r$ ]  $\implies$  is\_ub r x y u"

*<proof>*

**lemma** *is\_ubD*:

"is\_ub r x y u  $\implies$   $(x,u) \in r \wedge (y,u) \in r$ "

*<proof>*

```

lemma is_lub_bigger1 [iff]:
  "is_lub (r*) x y y = ((x,y) ∈ r*)" <proof>
lemma is_lub_bigger2 [iff]:
  "is_lub (r*) x y x = ((y,x) ∈ r*)" <proof>
lemma extend_lub:
  "[ single_valued r; is_lub (r*) x y u; (x',x) ∈ r ]
  ⇒ ∃ v. is_lub (r*) x' y v" <proof>
lemma single_valued_has_lubs [rule_format]:
  "[single_valued r; (x,u) ∈ r*] ⇒ (∀ y. (y,u) ∈ r* →
  (∃ z. is_lub (r*) x y z))" <proof>
lemma some_lub_conv:
  "[acyclic r; is_lub (r*) x y u] ⇒ some_lub (r*) x y = u" <proof>
lemma is_lub_some_lub:
  "[single_valued r; acyclic r; (x,u) ∈ r*; (y,u) ∈ r*]
  ⇒ is_lub (r*) x y (some_lub (r*) x y)"
  <proof>

```

#### 4.1.1 An executable lub-finder

**definition** `exec_lub` :: `"('a * 'a) set ⇒ ('a ⇒ 'a) ⇒ 'a binop"` where  
`"exec_lub r f x y ≡ while (λz. (x,z) ∉ r*) f y"`

```

lemma exec_lub_refl: "exec_lub r f T T = T"
<proof>

```

```

lemma acyclic_single_valued_finite:
  "[acyclic r; single_valued r; (x,y) ∈ r*]
  ⇒ finite (r ∩ {a. (x, a) ∈ r*} × {b. (b, y) ∈ r*})" <proof>

```

```

lemma exec_lub_conv:
  "[ acyclic r; ∀ x y. (x,y) ∈ r → f x = y; is_lub (r*) x y u ] ⇒
  exec_lub r f x y = u" <proof>

```

```

lemma is_lub_exec_lub:
  "[ single_valued r; acyclic r; (x,u) ∈ r*; (y,u) ∈ r*; ∀ x y. (x,y) ∈ r → f x = y ]
  ⇒ is_lub (r*) x y (exec_lub r f x y)"
  <proof>
end

```

## 4.2 The Error Type

```

theory Err
imports Semilat
begin

```

```

datatype 'a err = Err | OK 'a

```

```

type_synonym 'a ebinop = "'a ⇒ 'a ⇒ 'a err"
type_synonym 'a esl = "'a set * 'a ord * 'a ebinop"

```

```

primrec ok_val :: "'a err ⇒ 'a" where
  "ok_val (OK x) = x"

```

**definition** `lift` :: `"('a ⇒ 'b err) ⇒ ('a err ⇒ 'b err)"` where

```
"lift f e == case e of Err ⇒ Err | OK x ⇒ f x"
```

```
definition lift2 :: "('a ⇒ 'b ⇒ 'c err) ⇒ 'a err ⇒ 'b err ⇒ 'c err" where
"lift2 f e1 e2 ==
  case e1 of Err ⇒ Err
           | OK x ⇒ (case e2 of Err ⇒ Err | OK y ⇒ f x y)"
```

```
definition le :: "'a ord ⇒ 'a err ord" where
"le r e1 e2 ==
  case e2 of Err ⇒ True |
           OK y ⇒ (case e1 of Err ⇒ False | OK x ⇒ x <=_r y)"
```

```
definition sup :: "('a ⇒ 'b ⇒ 'c) ⇒ ('a err ⇒ 'b err ⇒ 'c err)" where
"sup f == lift2(%x y. OK(x +_f y))"
```

```
definition err :: "'a set ⇒ 'a err set" where
"err A == insert Err {x . ∃y∈A. x = OK y}"
```

```
definition esl :: "'a sl ⇒ 'a esl" where
"esl == %(A,r,f). (A,r, %x y. OK(f x y))"
```

```
definition sl :: "'a esl ⇒ 'a err sl" where
"sl == %(A,r,f). (err A, le r, lift2 f)"
```

```
abbreviation
  err_semlat :: "'a esl ⇒ bool"
  where "err_semlat L == semlat(Err.sl L)"
```

```
primrec strict :: "('a ⇒ 'b err) ⇒ ('a err ⇒ 'b err)" where
  "strict f Err = Err"
| "strict f (OK x) = f x"
```

```
lemma strict_Some [simp]:
  "(strict f x = OK y) = (∃ z. x = OK z ∧ f z = OK y)"
  <proof>
```

```
lemma not_Err_eq:
  "(x ≠ Err) = (∃ a. x = OK a)"
  <proof>
```

```
lemma not_OK_eq:
  "(∀ y. x ≠ OK y) = (x = Err)"
  <proof>
```

```
lemma unfold_le_sub_err:
  "e1 <=_ (le r) e2 == le r e1 e2"
  <proof>
```

```
lemma le_err_refl:
  "∀ x. x <=_r x ⇒ e <=_ (Err.le r) e"
  <proof>
```

```
lemma le_err_trans [rule_format]:
```

"order  $r \implies e1 \leq_{(le\ r)} e2 \longrightarrow e2 \leq_{(le\ r)} e3 \longrightarrow e1 \leq_{(le\ r)} e3$ "  
 <proof>

lemma le\_err\_antisym [rule\_format]:  
 "order  $r \implies e1 \leq_{(le\ r)} e2 \longrightarrow e2 \leq_{(le\ r)} e1 \longrightarrow e1=e2$ "  
 <proof>

lemma OK\_le\_err\_OK:  
 "(OK  $x \leq_{(le\ r)} OK\ y = (x \leq_{_r} y)$ "  
 <proof>

lemma order\_le\_err [iff]:  
 "order( $le\ r$ ) = order  $r$ "  
 <proof>

lemma le\_Err [iff]: " $e \leq_{(le\ r)} Err$ "  
 <proof>

lemma Err\_le\_conv [iff]:  
 " $Err \leq_{(le\ r)} e = (e = Err)$ "  
 <proof>

lemma le\_OK\_conv [iff]:  
 " $e \leq_{(le\ r)} OK\ x = (\exists y. e = OK\ y \ \&\ y \leq_{_r} x)$ "  
 <proof>

lemma OK\_le\_conv:  
 " $OK\ x \leq_{(le\ r)} e = (e = Err \mid (\exists y. e = OK\ y \ \&\ x \leq_{_r} y))$ "  
 <proof>

lemma top\_Err [iff]: "top ( $le\ r$ ) Err"  
 <proof>

lemma OK\_less\_conv [rule\_format, iff]:  
 " $OK\ x <_{(le\ r)} e = (e=Err \mid (\exists y. e = OK\ y \ \&\ x <_{_r} y))$ "  
 <proof>

lemma not\_Err\_less [rule\_format, iff]:  
 " $\sim (Err <_{(le\ r)} x)$ "  
 <proof>

lemma semilat\_errI [intro]:  
 assumes semilat: "semilat ( $A, r, f$ )"  
 shows "semilat(err  $A, Err.le\ r, lift2(\%x\ y. OK(f\ x\ y)))$ "  
 <proof>

lemma err\_semilat\_eslI\_aux:  
 assumes semilat: "semilat ( $A, r, f$ )"  
 shows "err\_semilat(esl( $A, r, f$ ))"  
 <proof>

lemma err\_semilat\_eslI [intro, simp]:  
 " $\bigwedge L. semilat\ L \implies err\_semilat(esl\ L)$ "  
 <proof>

**lemma** `acc_err [simp, intro!]: "acc r  $\implies$  acc(le r)"`  
`<proof>`

**lemma** `Err_in_err [iff]: "Err  $\in$  err A"`  
`<proof>`

**lemma** `Ok_in_err [iff]: "(Ok x  $\in$  err A) = (x $\in$ A)"`  
`<proof>`

#### 4.2.1 lift

**lemma** `lift_in_errI:`  
`"[ e  $\in$  err S;  $\forall x \in S. e = Ok\ x \longrightarrow f\ x \in err\ S ] \implies lift\ f\ e \in err\ S"$`   
`<proof>`

**lemma** `Err_lift2 [simp]:`  
`"Err +_(lift2 f) x = Err"`  
`<proof>`

**lemma** `lift2_Err [simp]:`  
`"x +_(lift2 f) Err = Err"`  
`<proof>`

**lemma** `Ok_lift2_Ok [simp]:`  
`"Ok x +_(lift2 f) Ok y = x +_f y"`  
`<proof>`

#### 4.2.2 sup

**lemma** `Err_sup_Err [simp]:`  
`"Err +_(Err.sup f) x = Err"`  
`<proof>`

**lemma** `Err_sup_Err2 [simp]:`  
`"x +_(Err.sup f) Err = Err"`  
`<proof>`

**lemma** `Err_sup_Ok [simp]:`  
`"Ok x +_(Err.sup f) Ok y = Ok(x +_f y)"`  
`<proof>`

**lemma** `Err_sup_eq_Ok_conv [iff]:`  
`"(Err.sup f ex ey = Ok z) = ( $\exists x\ y. ex = Ok\ x \ \&\ ey = Ok\ y \ \&\ f\ x\ y = z$ )"`  
`<proof>`

**lemma** `Err_sup_eq_Err [iff]:`  
`"(Err.sup f ex ey = Err) = (ex=Err | ey=Err)"`  
`<proof>`

#### 4.2.3 semilat (err A) (le r) f

**lemma** `semilat_le_err_Err_plus [simp]:`  
`"[ x  $\in$  err A; semilat(err A, le r, f) ]  $\implies Err +_f x = Err$ "`  
`<proof>`

```
lemma semilat_le_err_plus_Err [simp]:
  "[[ x ∈ err A; semilat(err A, le r, f) ]] ⇒ x +_f Err = Err"
  ⟨proof⟩
```

```
lemma semilat_le_err_OK1:
  "[[ x ∈ A; y ∈ A; semilat(err A, le r, f); OK x +_f OK y = OK z ]]
   ⇒ x <=_r z"
  ⟨proof⟩
```

```
lemma semilat_le_err_OK2:
  "[[ x ∈ A; y ∈ A; semilat(err A, le r, f); OK x +_f OK y = OK z ]]
   ⇒ y <=_r z"
  ⟨proof⟩
```

```
lemma eq_order_le:
  "[[ x=y; order r ]] ⇒ x <=_r y"
  ⟨proof⟩
```

```
lemma OK_plus_OK_eq_Err_conv [simp]:
  assumes "x ∈ A" and "y ∈ A" and "semilat(err A, le r, fe)"
  shows "((OK x) +_fe (OK y) = Err) = (¬(∃z∈A. x <=_r z & y <=_r z))"
  ⟨proof⟩
```

#### 4.2.4 semilat (err (Union AS))

```
lemma all_bex_swap_lemma [iff]:
  "(∀x. (∃y∈A. x = f y) → P x) = (∀y∈A. P(f y))"
  ⟨proof⟩
```

```
lemma closed_err_Union_lift2I:
  "[[ ∀A∈AS. closed (err A) (lift2 f); AS ≠ {};
    ∀A∈AS. ∀B∈AS. A≠B → (∀a∈A. ∀b∈B. a +_f b = Err) ]]
   ⇒ closed (err (⋃ AS)) (lift2 f)"
  ⟨proof⟩
```

If  $AS = \{\}$  the thm collapses to  $order\ r \wedge closed\ \{Err\}\ f \wedge Err \sqcup_f Err = Err$  which may not hold

```
lemma err_semilat_UnionI:
  "[[ ∀A∈AS. err_semilat(A, r, f); AS ≠ {};
    ∀A∈AS. ∀B∈AS. A≠B → (∀a∈A. ∀b∈B. ¬ a <=_r b & a +_f b = Err) ]]
   ⇒ err_semilat (⋃ AS, r, f)"
  ⟨proof⟩
```

end

### 4.3 Fixed Length Lists

```
theory Listn
imports Err
begin
```

```
definition list :: "nat ⇒ 'a set ⇒ 'a list set" where
```

```
"list n A == {xs. length xs = n & set xs <= A}"
```

```
definition le :: "'a ord ⇒ ('a list)ord" where
"le r == list_all2 (%x y. x <=_r y)"
```

abbreviation

```
lesublist_syntax :: "'a list ⇒ 'a ord ⇒ 'a list ⇒ bool"
  (<(_ /<=[_] _)> [50, 0, 51] 50)
where "x <=[r] y == x <_(le r) y"
```

abbreviation

```
lessublist_syntax :: "'a list ⇒ 'a ord ⇒ 'a list ⇒ bool"
  (<(_ /<[_] _)> [50, 0, 51] 50)
where "x <[r] y == x <_(le r) y"
```

```
definition map2 :: "('a ⇒ 'b ⇒ 'c) ⇒ 'a list ⇒ 'b list ⇒ 'c list" where
"map2 f == (%xs ys. map (case_prod f) (zip xs ys))"
```

abbreviation

```
plussublist_syntax :: "'a list ⇒ ('a ⇒ 'b ⇒ 'c) ⇒ 'b list ⇒ 'c list"
  (<(_ /+[_] _)> [65, 0, 66] 65)
where "x +[f] y == x +_(map2 f) y"
```

```
primrec coalesce :: "'a err list ⇒ 'a list err" where
  "coalesce [] = OK[]"
| "coalesce (ex#exs) = Err.sup (#) ex (coalesce exs)"
```

```
definition sl :: "nat ⇒ 'a sl ⇒ 'a list sl" where
"sl n == %(A,r,f). (list n A, le r, map2 f)"
```

```
definition sup :: "('a ⇒ 'b ⇒ 'c err) ⇒ 'a list ⇒ 'b list ⇒ 'c list err" where
"sup f == %xs ys. if size xs = size ys then coalesce(xs +[f] ys) else Err"
```

```
definition upto_esl :: "nat ⇒ 'a esl ⇒ 'a list esl" where
"upto_esl m == %(A,r,f). (⋃ {list n A | n. n <= m}, le r, sup f)"
```

```
lemmas [simp] = set_update_subsetI
```

```
lemma unfold_lesub_list:
  "xs <=[r] ys == Listn.le r xs ys"
  <proof>
```

```
lemma Nil_le_conv [iff]:
  "([ ] <=[r] ys) = (ys = [ ])"
  <proof>
```

```
lemma Cons_notle_Nil [iff]:
  "~ x#xs <=[r] [ ]"
  <proof>
```

```
lemma Cons_le_Cons [iff]:
  "x#xs <=[r] y#ys = (x <=_r y & xs <=[r] ys)"
  <proof>
```

**lemma** *Cons\_less\_Conss* [simp]:

"order r  $\implies$   
 $x\#xs <_{(Listn.le\ r)} y\#ys =$   
 $(x <_r y \ \& \ xs \leq[r] ys \mid x = y \ \& \ xs <_{(Listn.le\ r)} ys)$ "  
 <proof>

**lemma** *list\_update\_le\_cong*:

"[ i < size xs; xs  $\leq[r]$  ys; x  $\leq_r$  y ]  $\implies$  xs[i:=x]  $\leq[r]$  ys[i:=y]"  
 <proof>

**lemma** *le\_listD*:

"[ xs  $\leq[r]$  ys; p < size xs ]  $\implies$  xs!p  $\leq_r$  ys!p"  
 <proof>

**lemma** *le\_list\_refl*:

" $\forall x. x \leq_r x \implies xs \leq[r] xs$ "  
 <proof>

**lemma** *le\_list\_trans*:

"[ order r; xs  $\leq[r]$  ys; ys  $\leq[r]$  zs ]  $\implies$  xs  $\leq[r]$  zs"  
 <proof>

**lemma** *le\_list\_antisym*:

"[ order r; xs  $\leq[r]$  ys; ys  $\leq[r]$  xs ]  $\implies$  xs = ys"  
 <proof>

**lemma** *order\_listI* [simp, intro!]:

"order r  $\implies$  order(Listn.le r)"  
 <proof>

**lemma** *lesub\_list\_impl\_same\_size* [simp]:

"xs  $\leq[r]$  ys  $\implies$  size ys = size xs"  
 <proof>

**lemma** *lesssub\_list\_impl\_same\_size*:

"xs <\_{(Listn.le\ r)} ys  $\implies$  size ys = size xs"  
 <proof>

**lemma** *le\_list\_appendI*:

" $\bigwedge b\ c\ d. a \leq[r] b \implies c \leq[r] d \implies a@c \leq[r] b@d$ "  
 <proof>

**lemma** *le\_listI*:

"length a = length b  $\implies$  ( $\bigwedge n. n < \text{length } a \implies a!n \leq_r b!n$ )  $\implies$  a  $\leq[r]$  b"  
 <proof>

**lemma** *listI*:

"[ length xs = n; set xs  $\leq$  A ]  $\implies$  xs  $\in$  list n A"  
 <proof>

**lemma** *listE\_length* [simp]:



" $xs \in \text{list } n \ A \implies \text{length } xs = n$ "  
 $\langle \text{proof} \rangle$

**lemma less\_lengthI:**  
 " $\llbracket xs \in \text{list } n \ A; p < n \rrbracket \implies p < \text{length } xs$ "  
 $\langle \text{proof} \rangle$

**lemma listE\_set [simp]:**  
 " $xs \in \text{list } n \ A \implies \text{set } xs \leq A$ "  
 $\langle \text{proof} \rangle$

**lemma list\_0 [simp]:**  
 " $\text{list } 0 \ A = \{[]\}$ "  
 $\langle \text{proof} \rangle$

**lemma in\_list\_Suc\_iff:**  
 " $(xs \in \text{list } (\text{Suc } n) \ A) = (\exists y \in A. \exists ys \in \text{list } n \ A. xs = y \# ys)$ "  
 $\langle \text{proof} \rangle$

**lemma Cons\_in\_list\_Suc [iff]:**  
 " $(x \# xs \in \text{list } (\text{Suc } n) \ A) = (x \in A \ \& \ xs \in \text{list } n \ A)$ "  
 $\langle \text{proof} \rangle$

**lemma list\_not\_empty:**  
 " $\exists a. a \in A \implies \exists xs. xs \in \text{list } n \ A$ "  
 $\langle \text{proof} \rangle$

**lemma nth\_in [rule\_format, simp]:**  
 " $\forall i \ n. \text{length } xs = n \longrightarrow \text{set } xs \leq A \longrightarrow i < n \longrightarrow (xs!i) \in A$ "  
 $\langle \text{proof} \rangle$

**lemma listE\_nth\_in:**  
 " $\llbracket xs \in \text{list } n \ A; i < n \rrbracket \implies (xs!i) \in A$ "  
 $\langle \text{proof} \rangle$

**lemma listn\_Cons\_Suc [elim!]:**  
 " $l \# xs \in \text{list } n \ A \implies (\bigwedge n'. n = \text{Suc } n' \implies l \in A \implies xs \in \text{list } n' \ A \implies P) \implies P$ "  
 $\langle \text{proof} \rangle$

**lemma listn\_appendE [elim!]:**  
 " $a \ @ \ b \in \text{list } n \ A \implies (\bigwedge n1 \ n2. n = n1 + n2 \implies a \in \text{list } n1 \ A \implies b \in \text{list } n2 \ A \implies P) \implies P$ "  
 $\langle \text{proof} \rangle$

**lemma listt\_update\_in\_list [simp, intro!]:**  
 " $\llbracket xs \in \text{list } n \ A; x \in A \rrbracket \implies xs[i := x] \in \text{list } n \ A$ "  
 $\langle \text{proof} \rangle$

**lemma plus\_list\_Nil [simp]:**  
 " $[] \ + \ [f] \ xs = []$ "  
 $\langle \text{proof} \rangle$

**lemma** *plus\_list\_Cons* [*simp*]:  
 "(*x#xs*) +[*f*] *ys* = (case *ys* of []  $\Rightarrow$  [] | *y#ys*  $\Rightarrow$  (*x* +<sub>f</sub> *y*)#(*xs* +[*f*] *ys*))"  
 <*proof*>

**lemma** *length\_plus\_list* [*rule\_format*, *simp*]:  
 " $\forall$  *ys*. *length*(*xs* +[*f*] *ys*) = min(*length xs*) (*length ys*)"  
 <*proof*>

**lemma** *nth\_plus\_list* [*rule\_format*, *simp*]:  
 " $\forall$  *xs ys i*. *length xs* = *n*  $\longrightarrow$  *length ys* = *n*  $\longrightarrow$  *i* < *n*  $\longrightarrow$   
 (*xs* +[*f*] *ys*)! *i* = (*xs*! *i*) +<sub>f</sub> (*ys*! *i*)"  
 <*proof*>

**lemma** (in *Semilat*) *plus\_list\_ub1* [*rule\_format*]:  
 "[ set *xs* <= *A*; set *ys* <= *A*; size *xs* = size *ys* ]  
 $\implies$  *xs* <=[*r*] *xs* +[*f*] *ys*"  
 <*proof*>

**lemma** (in *Semilat*) *plus\_list\_ub2*:  
 "[set *xs* <= *A*; set *ys* <= *A*; size *xs* = size *ys* ]  
 $\implies$  *ys* <=[*r*] *xs* +[*f*] *ys*"  
 <*proof*>

**lemma** (in *Semilat*) *plus\_list\_lub* [*rule\_format*]:  
**shows** " $\forall$  *xs ys zs*. set *xs* <= *A*  $\longrightarrow$  set *ys* <= *A*  $\longrightarrow$  set *zs* <= *A*  
 $\longrightarrow$  size *xs* = *n* & size *ys* = *n*  $\longrightarrow$   
*xs* <=[*r*] *zs* & *ys* <=[*r*] *zs*  $\longrightarrow$  *xs* +[*f*] *ys* <=[*r*] *zs*"  
 <*proof*>

**lemma** (in *Semilat*) *list\_update\_incr* [*rule\_format*]:  
 "*x*  $\in$  *A*  $\implies$  set *xs* <= *A*  $\longrightarrow$   
 ( $\forall$  *i*. *i* < size *xs*  $\longrightarrow$  *xs* <=[*r*] *xs*[*i* := *x* +<sub>f</sub> *xs*! *i*])"  
 <*proof*>

**lemma** *acc\_le\_listI* [*intro!*]:  
 "[ order *r*; acc *r* ]  $\implies$  acc(*Listn.le r*)"  
 <*proof*>

**lemma** *closed\_listI*:  
 "closed *S f*  $\implies$  closed (*list n S*) (map2 *f*)"  
 <*proof*>

**lemma** *Listn\_sl\_aux*:  
**assumes** "*semilat* (*A*, *r*, *f*)" **shows** "*semilat* (*Listn.sl n* (*A*, *r*, *f*))"  
 <*proof*>

**lemma** *Listn\_sl*: " $\bigwedge$  *L*. *semilat L*  $\implies$  *semilat* (*Listn.sl n L*)"  
 <*proof*>

**lemma** *coalesce\_in\_err\_list* [*rule\_format*]:  
 " $\forall$  *xes*. *xes*  $\in$  *list n* (*err A*)  $\longrightarrow$  coalesce *xes*  $\in$  *err*(*list n A*)"

$\langle proof \rangle$

**lemma** lem: " $\bigwedge x \text{ xs. } x +_{\#} \text{ xs} = x\#\text{xs}$ "

$\langle proof \rangle$

**lemma** coalesce\_eq\_OK1\_D [rule\_format]:

"semilat(err A, Err.le r, lift2 f)  $\implies$   
 $\forall \text{xs. xs} \in \text{list } n \text{ A} \longrightarrow (\forall \text{ys. ys} \in \text{list } n \text{ A} \longrightarrow$   
 $(\forall \text{zs. coalesce (xs } +[f] \text{ ys)} = \text{OK zs} \longrightarrow \text{xs} \leq[r] \text{ zs}))$ "

$\langle proof \rangle$

**lemma** coalesce\_eq\_OK2\_D [rule\_format]:

"semilat(err A, Err.le r, lift2 f)  $\implies$   
 $\forall \text{xs. xs} \in \text{list } n \text{ A} \longrightarrow (\forall \text{ys. ys} \in \text{list } n \text{ A} \longrightarrow$   
 $(\forall \text{zs. coalesce (xs } +[f] \text{ ys)} = \text{OK zs} \longrightarrow \text{ys} \leq[r] \text{ zs}))$ "

$\langle proof \rangle$

**lemma** lift2\_le\_ub:

" $\llbracket \text{semilat(err A, Err.le r, lift2 f); } x \in \text{A}; y \in \text{A}; x +_f y = \text{OK } z;$   
 $u \in \text{A}; x \leq_r u; y \leq_r u \rrbracket \implies z \leq_r u$ "

$\langle proof \rangle$

**lemma** coalesce\_eq\_OK\_ub\_D [rule\_format]:

"semilat(err A, Err.le r, lift2 f)  $\implies$   
 $\forall \text{xs. xs} \in \text{list } n \text{ A} \longrightarrow (\forall \text{ys. ys} \in \text{list } n \text{ A} \longrightarrow$   
 $(\forall \text{zs us. coalesce (xs } +[f] \text{ ys)} = \text{OK zs} \wedge \text{xs} \leq[r] \text{ us} \wedge \text{ys} \leq[r] \text{ us}$   
 $\wedge \text{us} \in \text{list } n \text{ A} \longrightarrow \text{zs} \leq[r] \text{ us}))$ "

$\langle proof \rangle$

**lemma** lift2\_eq\_ErrD:

" $\llbracket x +_f y = \text{Err}; \text{semilat(err A, Err.le r, lift2 f); } x \in \text{A}; y \in \text{A} \rrbracket$   
 $\implies \sim(\exists u \in \text{A. } x \leq_r u \ \& \ y \leq_r u)$ "

$\langle proof \rangle$

**lemma** coalesce\_eq\_Err\_D [rule\_format]:

" $\llbracket \text{semilat(err A, Err.le r, lift2 f)} \rrbracket$   
 $\implies \forall \text{xs. xs} \in \text{list } n \text{ A} \longrightarrow (\forall \text{ys. ys} \in \text{list } n \text{ A} \longrightarrow$   
 $\text{coalesce (xs } +[f] \text{ ys)} = \text{Err} \longrightarrow$   
 $\neg(\exists \text{zs} \in \text{list } n \text{ A. } \text{xs} \leq[r] \text{ zs} \wedge \text{ys} \leq[r] \text{ zs}))$ "

$\langle proof \rangle$

**lemma** closed\_err\_lift2\_conv:

"closed (err A) (lift2 f) =  $(\forall x \in \text{A. } \forall y \in \text{A. } x +_f y \in \text{err A})$ "

$\langle proof \rangle$

**lemma** closed\_map2\_list [rule\_format]:

"closed (err A) (lift2 f)  $\implies$   
 $\forall \text{xs. xs} \in \text{list } n \text{ A} \longrightarrow (\forall \text{ys. ys} \in \text{list } n \text{ A} \longrightarrow$   
 $\text{map2 f xs ys} \in \text{list } n \text{ (err A)})$ "

$\langle proof \rangle$

**lemma** closed\_lift2\_sup:

"closed (err A) (lift2 f)  $\implies$

```

closed (err (list n A)) (lift2 (sup f))"
⟨proof⟩

lemma err_semilat_sup:
  "err_semilat (A,r,f) ==>
   err_semilat (list n A, Listn.le r, sup f)"
⟨proof⟩

lemma err_semilat_upto_esl:
  "ΛL. err_semilat L ==> err_semilat(upto_esl m L)"
⟨proof⟩

end

```

## 4.4 Typing and Dataflow Analysis Framework

```

theory Typing_Framework
imports Listn
begin

```

The relationship between dataflow analysis and a welltyped-instruction predicate.

```

type_synonym 's step_type = "nat ⇒ 's ⇒ (nat × 's) list"

definition stable :: "'s ord ⇒ 's step_type ⇒ 's list ⇒ nat ⇒ bool" where
  "stable r step ss p == ∀ (q,s')∈set(step p (ss!p)). s' <=_r ss!q"

definition stables :: "'s ord ⇒ 's step_type ⇒ 's list ⇒ bool" where
  "stables r step ss == ∀p<size ss. stable r step ss p"

definition wt_step ::
  "'s ord ⇒ 's ⇒ 's step_type ⇒ 's list ⇒ bool" where
  "wt_step r T step ts ==
   ∀p<size(ts). ts!p ~= T & stable r step ts p"

definition is_bcv :: "'s ord ⇒ 's ⇒ 's step_type
  ⇒ nat ⇒ 's set ⇒ ('s list ⇒ 's list) ⇒ bool" where
  "is_bcv r T step n A bcv == ∀ss ∈ list n A.
   (∀p<n. (bcv ss)!p ~= T) =
   (∃ts ∈ list n A. ss <=[r] ts & wt_step r T step ts)"

```

end

## 4.5 Products as Semilattices

```

theory Product
imports Err
begin

```

```

definition le :: "'a ord ⇒ 'b ord ⇒ ('a * 'b) ord" where
  "le rA rB == %(a,b) (a',b'). a <=_rA a' & b <=_rB b'"

definition sup :: "'a ebinop ⇒ 'b ebinop ⇒ ('a * 'b)ebinop" where
  "sup f g == %(a1,b1)(a2,b2). Err.sup Pair (a1+_f a2) (b1+_g b2)"

```

**definition** `esl` :: "'a esl  $\Rightarrow$  'b esl  $\Rightarrow$  ('a \* 'b) esl" **where**  
`"esl ==  $\%(A, rA, fA) (B, rB, fB). (A \times B, \text{le } rA \text{ } rB, \text{sup } fA \text{ } fB)$ "`

**abbreviation**

`lesubprod_syntax` :: "'a \* 'b  $\Rightarrow$  'a ord  $\Rightarrow$  'b ord  $\Rightarrow$  'a \* 'b  $\Rightarrow$  bool"  
`( $\langle$  _ /<= '(_,'_) _  $\rangle$  [50, 0, 0, 51] 50)`  
**where** `"p <=(rA,rB) q == p <=_ (le rA rB) q"`

**lemma** `unfold_lesub_prod`:

`"p <=(rA,rB) q == le rA rB p q"`  
 $\langle$ proof $\rangle$

**lemma** `le_prod_Pair_conv` [iff]:

`"((a1,b1) <=(rA,rB) (a2,b2)) = (a1 <=_rA a2 & b1 <=_rB b2)"`  
 $\langle$ proof $\rangle$

**lemma** `less_prod_Pair_conv`:

`"((a1,b1) <_ (Product.le rA rB) (a2,b2)) =  
(a1 <_rA a2 & b1 <=_rB b2 | a1 <=_rA a2 & b1 <_rB b2)"`  
 $\langle$ proof $\rangle$

**lemma** `order_le_prod` [iff]:

`"order(Product.le rA rB) = (order rA & order rB)"`  
 $\langle$ proof $\rangle$

**lemma** `acc_le_prodI` [intro!]:

`"[ acc rA; acc rB ]  $\implies$  acc(Product.le rA rB)"`  
 $\langle$ proof $\rangle$

**lemma** `closed_lift2_sup`:

`"[ closed (err A) (lift2 f); closed (err B) (lift2 g) ]  $\implies$   
closed (err(A $\times$ B)) (lift2(sup f g))"`  
 $\langle$ proof $\rangle$

**lemma** `unfold_plussub_lift2`:

`"e1 +_ (lift2 f) e2 == lift2 f e1 e2"`  
 $\langle$ proof $\rangle$

**lemma** `plus_eq_Err_conv` [simp]:

`assumes "x  $\in$  A" and "y  $\in$  A"  
and "semilat(err A, Err.le r, lift2 f)"  
shows "(x +_f y = Err) = ( $\neg(\exists z \in A. x <=_r z \& y <=_r z)$ )"`  
 $\langle$ proof $\rangle$

**lemma** `err_semilat_Product_esl`:

`" $\bigwedge$  L1 L2. [ err_semilat L1; err_semilat L2 ]  $\implies$  err_semilat(Product.esl L1 L2)"`  
 $\langle$ proof $\rangle$

**end**

## 4.6 More on Semilattices

```

theory SemilatAlg
imports Typing_Framework Product
begin

definition lesubstep_type :: "(nat × 's) list ⇒ 's ord ⇒ (nat × 's) list ⇒ bool"
  (<(_ /≤|_ | _)> [50, 0, 51] 50) where
  "x ≤|r| y ≡ ∀ (p,s) ∈ set x. ∃ s'. (p,s') ∈ set y ∧ s ≤_r s'"

primrec plusplussub :: "'a list ⇒ ('a ⇒ 'a ⇒ 'a) ⇒ 'a ⇒ 'a" (<(_ /++'__ _)> [65,
1000, 66] 65) where
  "[] ++_f y = y"
| "(x#xs) ++_f y = xs ++_f (x +_f y)"

definition bounded :: "'s step_type ⇒ nat ⇒ bool" where
  "bounded step n == ∀ p<n. ∀ s. ∀ (q,t)∈set(step p s). q<n"

definition pres_type :: "'s step_type ⇒ nat ⇒ 's set ⇒ bool" where
  "pres_type step n A == ∀ s∈A. ∀ p<n. ∀ (q,s')∈set (step p s). s' ∈ A"

definition mono :: "'s ord ⇒ 's step_type ⇒ nat ⇒ 's set ⇒ bool" where
  "mono r step n A ==
  ∀ s p t. s ∈ A ∧ p < n ∧ s ≤_r t ⟶ step p s ≤|r| step p t"

lemma pres_typeD:
  "[ pres_type step n A; s∈A; p<n; (q,s')∈set (step p s) ] ⟹ s' ∈ A"
  <proof>

lemma monoD:
  "[ mono r step n A; p < n; s∈A; s ≤_r t ] ⟹ step p s ≤|r| step p t"
  <proof>

lemma boundedD:
  "[ bounded step n; p < n; (q,t) ∈ set (step p xs) ] ⟹ q < n"
  <proof>

lemma lesubstep_type_refl [simp, intro]:
  "(∧ x. x ≤_r x) ⟹ x ≤|r| x"
  <proof>

lemma lesub_step_typeD:
  "a ≤|r| b ⟹ (x,y) ∈ set a ⟹ ∃ y'. (x, y') ∈ set b ∧ y ≤_r y'"
  <proof>

lemma list_update_le_listI [rule_format]:
  "set xs ≤ A ⟶ set ys ≤ A ⟶ xs ≤[r] ys ⟶ p < size xs ⟶
  x ≤_r ys!p ⟶ semilat(A,r,f) ⟶ x∈A ⟶
  xs[p := x +_f xs!p] ≤[r] ys"
  <proof>

lemma plusplus_closed: assumes "semilat (A, r, f)" shows

```

```
" $\bigwedge y. \llbracket \text{set } x \subseteq A; y \in A \rrbracket \implies x \text{ ++\_f } y \in A$ " (is "PROP ?P")
<proof>
```

```
lemma (in Semilat) pp_ub2:
  " $\bigwedge y. \llbracket \text{set } x \subseteq A; y \in A \rrbracket \implies y \leq_r x \text{ ++\_f } y$ "
<proof>
```

```
lemma (in Semilat) pp_ub1:
  shows " $\bigwedge y. \llbracket \text{set } ls \subseteq A; y \in A; x \in \text{set } ls \rrbracket \implies x \leq_r ls \text{ ++\_f } y$ "
<proof>
```

```
lemma (in Semilat) pp_lub:
  assumes z: "z  $\in$  A"
  shows
    " $\bigwedge y. y \in A \implies \text{set } xs \subseteq A \implies \forall x \in \text{set } xs. x \leq_r z \implies y \leq_r z \implies xs \text{ ++\_f } y \leq_r z$ "
<proof>
```

```
lemma ub1':
  assumes "semilat (A, r, f)"
  shows " $\llbracket \forall (p,s) \in \text{set } S. s \in A; y \in A; (a,b) \in \text{set } S \rrbracket$ 
 $\implies b \leq_r \text{map snd } [(p', t') \leftarrow S. p' = a] \text{ ++\_f } y$ "
<proof>
```

```
lemma plusplus_empty:
  " $\forall s'. (q, s') \in \text{set } S \longrightarrow s' \text{ ++\_f } ss ! q = ss ! q \implies$ 
 $(\text{map snd } [(p', t') \leftarrow S. p' = q] \text{ ++\_f } ss ! q) = ss ! q$ "
<proof>
```

```
end
```

## 4.7 Lifting the Typing Framework to err, app, and eff

```
theory Typing_Framework_err
imports Typing_Framework SemilatAlg
begin
```

```
definition wt_err_step :: "'s ord  $\Rightarrow$  's err step_type  $\Rightarrow$  's err list  $\Rightarrow$  bool" where
  "wt_err_step r step ts  $\equiv$  wt_step (Err.le r) Err step ts"
```

```
definition wt_app_eff :: "'s ord  $\Rightarrow$  (nat  $\Rightarrow$  's  $\Rightarrow$  bool)  $\Rightarrow$  's step_type  $\Rightarrow$  's list  $\Rightarrow$  bool"
where
  "wt_app_eff r app step ts  $\equiv$ 
 $\forall p < \text{size } ts. \text{app } p (ts!p) \wedge (\forall (q,t) \in \text{set } (\text{step } p (ts!p)). t \leq_r ts!q)$ "
```

```
definition map_snd :: "('b  $\Rightarrow$  'c)  $\Rightarrow$  ('a  $\times$  'b) list  $\Rightarrow$  ('a  $\times$  'c) list" where
  "map_snd f  $\equiv$  map ( $\lambda(x,y). (x, f y)$ )"
```

```
definition error :: "nat  $\Rightarrow$  (nat  $\times$  'a err) list" where
```

"error n  $\equiv$  map ( $\lambda x. (x, \text{Err})$ ) [0.. $n$ ]"

**definition** err\_step :: "nat  $\Rightarrow$  (nat  $\Rightarrow$  's  $\Rightarrow$  bool)  $\Rightarrow$  's step\_type  $\Rightarrow$  's err\_step\_type"  
**where**

"err\_step n app step p t  $\equiv$   
 case t of  
   Err  $\Rightarrow$  error n  
   | OK t'  $\Rightarrow$  if app p t' then map\_snd OK (step p t') else error n"

**definition** app\_mono :: "'s ord  $\Rightarrow$  (nat  $\Rightarrow$  's  $\Rightarrow$  bool)  $\Rightarrow$  nat  $\Rightarrow$  's set  $\Rightarrow$  bool" **where**

"app\_mono r app n A  $\equiv$   
 $\forall s p t. s \in A \wedge p < n \wedge s \leq_r t \longrightarrow \text{app } p \ t \longrightarrow \text{app } p \ s$ "

**lemmas** err\_step\_defs = err\_step\_def map\_snd\_def error\_def

**lemma** bounded\_err\_stepD:

"bounded (err\_step n app step) n  $\implies$   
 $p < n \implies \text{app } p \ a \implies (q, b) \in \text{set } (\text{step } p \ a) \implies$   
 $q < n$ "  
 <proof>

**lemma** in\_map\_sndD: "(a, b)  $\in$  set (map\_snd f xs)  $\implies \exists b'. (a, b') \in \text{set } xs$ "

<proof>

**lemma** bounded\_err\_stepI:

" $\forall p. p < n \longrightarrow (\forall s. \text{app } p \ s \longrightarrow (\forall (q, s') \in \text{set } (\text{step } p \ s). q < n))$   
 $\implies \text{bounded } (\text{err\_step } n \text{ app step}) \ n$ "  
 <proof>

**lemma** bounded\_lift:

"bounded step n  $\implies \text{bounded } (\text{err\_step } n \text{ app step}) \ n$ "  
 <proof>

**lemma** le\_list\_map\_OK [simp]:

" $\bigwedge b. \text{map } \text{OK } a \leq [\text{Err.le } r] \text{ map } \text{OK } b = (a \leq [r] b)$ "  
 <proof>

**lemma** map\_snd\_lessI:

" $x \leq |r| \ y \implies \text{map\_snd } \text{OK } x \leq [\text{Err.le } r] \text{ map\_snd } \text{OK } y$ "  
 <proof>

**lemma** mono\_lift:

"order r  $\implies \text{app\_mono } r \text{ app } n \ A \implies \text{bounded } (\text{err\_step } n \text{ app step}) \ n \implies$   
 $\forall s p t. s \in A \wedge p < n \wedge s \leq_r t \longrightarrow \text{app } p \ t \longrightarrow \text{step } p \ s \leq |r| \ \text{step } p \ t \implies$   
 $\text{mono } (\text{Err.le } r) \ (\text{err\_step } n \text{ app step}) \ n \ (\text{err } A)$ "  
 <proof>



**lemma** *in\_errorD*:

" $(x,y) \in \text{set } (\text{error } n) \implies y = \text{Err}$ "  
 <proof>

**lemma** *pres\_type\_lift*:

" $\forall s \in A. \forall p. p < n \longrightarrow \text{app } p \ s \longrightarrow (\forall (q, s') \in \text{set } (\text{step } p \ s). s' \in A)$   
 $\implies \text{pres\_type } (\text{err\_step } n \ \text{app } \text{step}) \ n \ (\text{err } A)$ "  
 <proof>

There used to be a condition here that each instruction must have a successor. This is not needed any more, because the definition of *error* trivially ensures that there is a successor for the critical case where *app* does not hold.

**lemma** *wt\_err\_imp\_wt\_app\_eff*:

assumes *wt*: "*wt\_err\_step* *r* (*err\_step* (*size ts*) *app step*) *ts*"  
 assumes *b*: "*bounded* (*err\_step* (*size ts*) *app step*) (*size ts*)"  
 shows "*wt\_app\_eff* *r* *app step* (*map ok\_val ts*)"  
 <proof>

**lemma** *wt\_app\_eff\_imp\_wt\_err*:

assumes *app\_eff*: "*wt\_app\_eff* *r* *app step* *ts*"  
 assumes *bounded*: "*bounded* (*err\_step* (*size ts*) *app step*) (*size ts*)"  
 shows "*wt\_err\_step* *r* (*err\_step* (*size ts*) *app step*) (*map OK ts*)"  
 <proof>

end

## 4.8 Kildall's Algorithm

**theory** *Kildall*

imports *SemilatAlg* "*HOL-Library.While\_Combinator*"

begin

**primrec** *propa* :: "'s binop  $\Rightarrow$  (nat  $\times$  's) list  $\Rightarrow$  's list  $\Rightarrow$  nat set  $\Rightarrow$  's list \* nat set" where

"*propa* *f* [] ss *w* = (*ss*,*w*)"  
 | "*propa* *f* (*q*'#*qs*) ss *w* = (let (*q*,*t*) = *q*';  
     *u* = *t* +<sub>f</sub> *ss*!*q*;  
     *w*' = (if *u* = *ss*!*q* then *w* else insert *q* *w*)  
   in *propa* *f* *qs* (*ss*[*q* := *u*] *w*')"

**definition** *iter* :: "'s binop  $\Rightarrow$  's step\_type  $\Rightarrow$  's list  $\Rightarrow$  nat set  $\Rightarrow$  's list  $\times$  nat set" where

"*iter* *f* *step* *ss* *w* == while (%(*ss*,*w*). *w*  $\neq$  {})  
 (%(*ss*,*w*). let *p* = *SOME* *p*. *p*  $\in$  *w*  
     in *propa* *f* (*step* *p* (*ss*!*p*)) *ss* (*w*-{*p*}))  
 (*ss*,*w*)"

**definition** *unstables* :: "'s ord  $\Rightarrow$  's step\_type  $\Rightarrow$  's list  $\Rightarrow$  nat set" where

"*unstables* *r* *step* *ss* == {*p*. *p* < *size ss*  $\wedge$   $\neg$ stable *r* *step* *ss* *p*}"

**definition** *kildall* :: "'s ord  $\Rightarrow$  's binop  $\Rightarrow$  's step\_type  $\Rightarrow$  's list  $\Rightarrow$  's list" where

```
"kildall r f step ss == fst(iter f step ss (unstables r step ss))"
```

```
primrec merges :: "'s binop  $\Rightarrow$  (nat  $\times$  's) list  $\Rightarrow$  's list  $\Rightarrow$  's list" where
  "merges f [] ss = ss"
| "merges f (p'#ps) ss = (let (p,s) = p' in merges f ps (ss[p := s+_f ss!p]))"
```

```
lemmas [simp] = Let_def Semilat.le_iff_plus_unchanged [OF Semilat.intro, symmetric]
```

```
lemma (in Semilat) nth_merges:
  " $\bigwedge ss. [p < \text{length } ss; ss \in \text{list } n \ A; \forall (p,t) \in \text{set } ps. p < n \wedge t \in A] \implies$ 
   $(\text{merges } f \ ps \ ss)!p = \text{map } \text{snd } [(p',t') \leftarrow ps. p'=p] \ ++_f \ ss!p$ "
  (is " $\bigwedge ss. [_; _; ?steptype \ ps] \implies ?P \ ss \ ps$ ")
<proof>
```

```
lemma length_merges [simp]: "size(merges f ps ss) = size ss"
  <proof>
```

```
lemma (in Semilat) merges_preserves_type_lemma:
  shows " $\forall xs. xs \in \text{list } n \ A \longrightarrow (\forall (p,x) \in \text{set } ps. p < n \wedge x \in A)$ 
   $\longrightarrow \text{merges } f \ ps \ xs \in \text{list } n \ A$ "
  <proof>
```

```
lemma (in Semilat) merges_preserves_type [simp]:
  " $[xs \in \text{list } n \ A; \forall (p,x) \in \text{set } ps. p < n \wedge x \in A]$ 
   $\implies \text{merges } f \ ps \ xs \in \text{list } n \ A$ "
  <proof>
```

```
lemma (in Semilat) merges_incr_lemma:
  " $\forall xs. xs \in \text{list } n \ A \longrightarrow (\forall (p,x) \in \text{set } ps. p < \text{size } xs \wedge x \in A) \longrightarrow xs \leq[r] \text{merges } f \ ps$ 
   $xs$ "
  <proof>
```

```
lemma (in Semilat) merges_incr:
  " $[xs \in \text{list } n \ A; \forall (p,x) \in \text{set } ps. p < \text{size } xs \wedge x \in A]$ 
   $\implies xs \leq[r] \text{merges } f \ ps \ xs$ "
  <proof>
```

```
lemma (in Semilat) merges_same_conv [rule_format]:
  " $(\forall xs. xs \in \text{list } n \ A \longrightarrow (\forall (p,x) \in \text{set } ps. p < \text{size } xs \wedge x \in A) \longrightarrow$ 
   $(\text{merges } f \ ps \ xs = xs) = (\forall (p,x) \in \text{set } ps. x \leq_r xs!p))$ "
  <proof>
```

```
lemma (in Semilat) list_update_le_listI [rule_format]:
  " $\text{set } xs \leq A \longrightarrow \text{set } ys \leq A \longrightarrow xs \leq[r] ys \longrightarrow p < \text{size } xs \longrightarrow$ 
   $x \leq_r ys!p \longrightarrow x \in A \longrightarrow xs[p := x+_f xs!p] \leq[r] ys$ "
  <proof>
```

```

lemma (in Semilat) merges_pres_le_ub:
  assumes "set ts <= A" and "set ss <= A"
    and " $\forall (p,t) \in \text{set } ps. t \leq_r ts!p \wedge t \in A \wedge p < \text{size } ts$ " and " $ss \leq[r] ts$ "
  shows "merges f ps ss <=[r] ts"
<proof>

```

```

lemma decomp_propa:
  " $\bigwedge ss \ w. (\forall (q,t) \in \text{set } qs. q < \text{size } ss) \implies$ "
  propa f qs ss w =
  (merges f qs ss, {q.  $\exists t. (q,t) \in \text{set } qs \wedge t +_f ss!q \neq ss!q$ } Un w)"
<proof>

```

```

lemma (in Semilat) stable_pres_lemma:
shows "[pres_type step n A; bounded step n;
  ss  $\in$  list n A; p  $\in$  w;  $\forall q \in w. q < n$ ;
   $\forall q. q < n \longrightarrow q \notin w \longrightarrow \text{stable } r \text{ step } ss \ q; q < n$ ;
   $\forall s'. (q,s') \in \text{set } (\text{step } p (ss ! p)) \longrightarrow s' +_f ss ! q = ss ! q$ ;
   $q \notin w \vee q = p$  ]
 $\implies \text{stable } r \text{ step } (\text{merges } f (\text{step } p (ss!p)) ss) \ q"$ 
<proof>

```

```

lemma (in Semilat) merges_bounded_lemma:
  "[ mono r step n A; bounded step n;
   $\forall (p',s') \in \text{set } (\text{step } p (ss!p)). s' \in A; ss \in \text{list } n \ A; ts \in \text{list } n \ A; p < n$ ;
   $ss \leq[r] ts; \forall p. p < n \longrightarrow \text{stable } r \text{ step } ts \ p$  ]
 $\implies \text{merges } f (\text{step } p (ss!p)) ss \leq[r] ts"$ 
<proof>

```

```

lemma termination_lemma:
  assumes semilat: "semilat (A, r, f)"
  shows "[ ss  $\in$  list n A;  $\forall (q,t) \in \text{set } qs. q < n \wedge t \in A; p \in w$  ]  $\implies$ 
   $ss <[r] \text{merges } f \ qs \ ss \vee$ 
   $\text{merges } f \ qs \ ss = ss \wedge \{q. \exists t. (q,t) \in \text{set } qs \wedge t +_f ss!q \neq ss!q\} \text{ Un } (w - \{p\}) < w$ " (is
  "PROP ?P")
<proof>

```

```

lemma iter_properties[rule_format]:
  assumes semilat: "semilat (A, r, f)"
  shows "[ acc r ; pres_type step n A; mono r step n A;
  bounded step n;  $\forall p \in w0. p < n; ss0 \in \text{list } n \ A$ ;
   $\forall p < n. p \notin w0 \longrightarrow \text{stable } r \text{ step } ss0 \ p$  ]  $\implies$ 
  iter f step ss0 w0 = (ss',w')
 $\longrightarrow$ 
   $ss' \in \text{list } n \ A \wedge \text{stables } r \text{ step } ss' \wedge ss0 \leq[r] ss' \wedge$ 
   $(\forall ts \in \text{list } n \ A. ss0 \leq[r] ts \wedge \text{stables } r \text{ step } ts \longrightarrow ss' \leq[r] ts)"$ 
  (is "PROP ?P")

```

*<proof>*

```
lemma kildall_properties:
  assumes semilat: "semilat (A, r, f)"
  shows "[[ acc r; pres_type step n A; mono r step n A;
    bounded step n; ss0 ∈ list n A ]] ⇒
    kildall r f step ss0 ∈ list n A ∧
    stables r step (kildall r f step ss0) ∧
    ss0 <=[r] kildall r f step ss0 ∧
    (∀ ts ∈ list n A. ss0 <=[r] ts ∧ stables r step ts →
      kildall r f step ss0 <=[r] ts)"
  (is "PROP ?P")
```

*<proof>*

```
lemma is_bcv_kildall:
  assumes semilat: "semilat (A, r, f)"
  shows "[[ acc r; top r T; pres_type step n A; bounded step n; mono r step n A ]]
    ⇒ is_bcv r T step n A (kildall r f step)"
  (is "PROP ?P")
```

*<proof>*

end

## 4.9 More about Options

```
theory Opt
  imports Err
  begin
```

```
definition le :: "'a ord ⇒ 'a option ord" where
  "le r o1 o2 == case o2 of None ⇒ o1=None |
    Some y ⇒ (case o1 of None ⇒ True
      | Some x ⇒ x <=_r y)"
```

```
definition opt :: "'a set ⇒ 'a option set" where
  "opt A == insert None {x. ∃ y ∈ A. x = Some y}"
```

```
definition sup :: "'a ebinop ⇒ 'a option ebinop" where
  "sup f o1 o2 ==
    case o1 of None ⇒ OK o2 | Some x ⇒ (case o2 of None ⇒ OK o1
      | Some y ⇒ (case f x y of Err ⇒ Err | OK z ⇒ OK (Some z)))"
```

```
definition esl :: "'a esl ⇒ 'a option esl" where
  "esl == %(A,r,f). (opt A, le r, sup f)"
```

```
lemma unfold_le_opt:
  "o1 <=_ (le r) o2 =
    (case o2 of None ⇒ o1=None |
      Some y ⇒ (case o1 of None ⇒ True | Some x ⇒ x <=_r y))"
```

*<proof>*

```
lemma le_opt_refl:
  "order r ⇒ o1 <=_ (le r) o1"
```

$\langle proof \rangle$

**lemma** `le_opt_trans [rule_format]:`

"order  $r \implies$

$o1 \leq (le\ r)\ o2 \longrightarrow o2 \leq (le\ r)\ o3 \longrightarrow o1 \leq (le\ r)\ o3$ "

$\langle proof \rangle$

**lemma** `le_opt_antisym [rule_format]:`

"order  $r \implies o1 \leq (le\ r)\ o2 \longrightarrow o2 \leq (le\ r)\ o1 \longrightarrow o1=o2$ "

$\langle proof \rangle$

**lemma** `order_le_opt [intro!,simp]:`

"order  $r \implies order(le\ r)$ "

$\langle proof \rangle$

**lemma** `None_bot [iff]:`

"None  $\leq (le\ r)\ ox$ "

$\langle proof \rangle$

**lemma** `Some_le [iff]:`

"(Some  $x \leq (le\ r)\ ox$ ) =  $(\exists y. ox = Some\ y \wedge x \leq_r y)$ "

$\langle proof \rangle$

**lemma** `le_None [iff]:`

"( $ox \leq (le\ r)\ None$ ) =  $(ox = None)$ "

$\langle proof \rangle$

**lemma** `OK_None_bot [iff]:`

"OK None  $\leq (Err.le\ (le\ r))\ x$ "

$\langle proof \rangle$

**lemma** `sup_None1 [iff]:`

" $x +_ (\sup\ f)\ None = OK\ x$ "

$\langle proof \rangle$

**lemma** `sup_None2 [iff]:`

"None  $+_ (\sup\ f)\ x = OK\ x$ "

$\langle proof \rangle$

**lemma** `None_in_opt [iff]:`

"None  $\in opt\ A$ "

$\langle proof \rangle$

**lemma** `Some_in_opt [iff]:`

"(Some  $x \in opt\ A$ ) =  $(x \in A)$ "

$\langle proof \rangle$

**lemma** `semilat_opt [intro, simp]:`

" $\bigwedge L. err\_semilat\ L \implies err\_semilat\ (Opt.esl\ L)$ "

$\langle proof \rangle$

```

lemma top_le_opt_Some [iff]:
  "top (le r) (Some T) = top r T"
<proof>

lemma Top_le_conv:
  "[[ order r; top r T ] => (T <=_r x) = (x = T)]"
<proof>

lemma acc_le_optI [intro!]:
  "acc r => acc(le r)"
<proof>

lemma option_map_in_optionI:
  "[[ ox ∈ opt S; ∀ x∈S. ox = Some x → f x ∈ S ]
   => map_option f ox ∈ opt S]"
<proof>

end

```

## 4.10 The Lightweight Bytecode Verifier

```

theory LBVSPEC
imports SemilatAlg Opt
begin

type_synonym 's certificate = "'s list"

primrec merge :: "'s certificate ⇒ 's binop ⇒ 's ord ⇒ 's ⇒ nat ⇒ (nat × 's) list
⇒ 's ⇒ 's" where
  "merge cert f r T pc [] x = x"
| "merge cert f r T pc (s#ss) x = merge cert f r T pc ss (let (pc',s') = s in
  if pc'=pc+1 then s' +_f x
  else if s' <=_r (cert!pc') then x
  else T)"

definition wtl_inst :: "'s certificate ⇒ 's binop ⇒ 's ord ⇒ 's ⇒
's step_type ⇒ nat ⇒ 's ⇒ 's" where
"wtl_inst cert f r T step pc s ≡ merge cert f r T pc (step pc s) (cert!(pc+1))"

definition wtl_cert :: "'s certificate ⇒ 's binop ⇒ 's ord ⇒ 's ⇒ 's ⇒
's step_type ⇒ nat ⇒ 's ⇒ 's" where
"wtl_cert cert f r T B step pc s ≡
  if cert!pc = B then
    wtl_inst cert f r T step pc s
  else
    if s <=_r (cert!pc) then wtl_inst cert f r T step pc (cert!pc) else T"

primrec wtl_inst_list :: "'a list ⇒ 's certificate ⇒ 's binop ⇒ 's ord ⇒ 's ⇒ 's
⇒
's step_type ⇒ nat ⇒ 's ⇒ 's" where
  "wtl_inst_list [] cert f r T B step pc s = s"
| "wtl_inst_list (i#is) cert f r T B step pc s =

```

```
(let s' = wtl_cert cert f r T B step pc s in
  if s' = T ∨ s = T then T else wtl_inst_list is cert f r T B step (pc+1) s')
```

**definition** cert\_ok :: "'s certificate ⇒ nat ⇒ 's ⇒ 's ⇒ 's set ⇒ bool" where  
 "cert\_ok cert n T B A ≡ (∀ i < n. cert!i ∈ A ∧ cert!i ≠ T) ∧ (cert!n = B)"

**definition** bottom :: "'a ord ⇒ 'a ⇒ bool" where  
 "bottom r B ≡ ∀ x. B ≤<sub>r</sub> x"

**locale** lbv = Semilat +  
 fixes T :: "'a" (<⊤>)  
 fixes B :: "'a" (<⊥>)  
 fixes step :: "'a step\_type"  
 assumes top: "top r ⊤"  
 assumes T\_A: "⊤ ∈ A"  
 assumes bot: "bottom r ⊥"  
 assumes B\_A: "⊥ ∈ A"

fixes merge :: "'a certificate ⇒ nat ⇒ (nat × 'a) list ⇒ 'a ⇒ 'a"  
 defines mrg\_def: "merge cert ≡ LBVSPEC.merge cert f r ⊤"

fixes wti :: "'a certificate ⇒ nat ⇒ 'a ⇒ 'a"  
 defines wti\_def: "wti cert ≡ wtl\_inst cert f r ⊤ step"

fixes wtc :: "'a certificate ⇒ nat ⇒ 'a ⇒ 'a"  
 defines wtc\_def: "wtc cert ≡ wtl\_cert cert f r ⊤ ⊥ step"

fixes wtl :: "'b list ⇒ 'a certificate ⇒ nat ⇒ 'a ⇒ 'a"  
 defines wtl\_def: "wtl ins cert ≡ wtl\_inst\_list ins cert f r ⊤ ⊥ step"

**lemma** (in lbv) wti:  
 "wti c pc s ≡ merge c pc (step pc s) (c!(pc+1))"  
 <proof>

**lemma** (in lbv) wtc:  
 "wtc c pc s ≡ if c!pc = ⊥ then wti c pc s else if s ≤<sub>r</sub> c!pc then wti c pc (c!pc)  
 else ⊤"  
 <proof>

**lemma** cert\_okD1 [intro]:  
 "cert\_ok c n T B A ⇒ pc < n ⇒ c!pc ∈ A"  
 <proof>

**lemma** cert\_okD2 [intro]:  
 "cert\_ok c n T B A ⇒ c!n = B"  
 <proof>

**lemma** cert\_okD3 [intro]:  
 "cert\_ok c n T B A ⇒ B ∈ A ⇒ pc < n ⇒ c!Suc pc ∈ A"  
 <proof>

**lemma** cert\_okD4 [intro]:

```
"cert_ok c n T B A  $\implies$  pc < n  $\implies$  c!pc  $\neq$  T"
<proof>
```

```
declare Let_def [simp]
```

#### 4.10.1 more semilattice lemmas

```
lemma (in lbv) sup_top [simp, elim]:
  assumes x: "x  $\in$  A"
  shows "x +_f  $\top$  =  $\top$ "
<proof>
```

```
lemma (in lbv) plusplussup_top [simp, elim]:
  "set xs  $\subseteq$  A  $\implies$  xs ++_f  $\top$  =  $\top$ "
<proof>
```

```
lemma (in Semilat) pp_ub1':
  assumes S: "snd'set S  $\subseteq$  A"
  assumes y: "y  $\in$  A" and ab: "(a, b)  $\in$  set S"
  shows "b <=_r map snd [(p', t')  $\leftarrow$  S . p' = a] ++_f y"
<proof>
```

```
lemma (in lbv) bottom_le [simp, intro]:
  " $\perp$  <=_r x"
<proof>
```

```
lemma (in lbv) le_bottom [simp]:
  "x <=_r  $\perp$  = (x =  $\perp$ )"
<proof>
```

#### 4.10.2 merge

```
lemma (in lbv) merge_Nil [simp]:
  "merge c pc [] x = x" <proof>
```

```
lemma (in lbv) merge_Cons [simp]:
  "merge c pc (l#ls) x = merge c pc ls (if fst l=pc+1 then snd l +_f x
                                           else if snd l <=_r (c!fst l) then x
                                           else  $\top$ )"
<proof>
```

```
lemma (in lbv) merge_Err [simp]:
  "snd'set ss  $\subseteq$  A  $\implies$  merge c pc ss  $\top$  =  $\top$ "
<proof>
```

```
lemma (in lbv) merge_not_top:
  " $\bigwedge$ x. snd'set ss  $\subseteq$  A  $\implies$  merge c pc ss x  $\neq$   $\top \implies$ 
 $\forall$  (pc', s')  $\in$  set ss. (pc'  $\neq$  pc+1  $\longrightarrow$  s' <=_r (c!pc'))"
  (is " $\bigwedge$ x. ?set ss  $\implies$  ?merge ss x  $\implies$  ?P ss")
<proof>
```



```

lemma (in lbv) merge_def:
  shows
    " $\bigwedge x. x \in A \implies \text{snd}' \text{set } ss \subseteq A \implies$ 
    merge c pc ss x =
    (if  $\forall (pc', s') \in \text{set } ss. pc' \neq pc+1 \longrightarrow s' \leq_r c!pc'$  then
      map snd [(p', t')  $\leftarrow$  ss. p'=pc+1] ++_f x
    else  $\top$ )"
    (is " $\bigwedge x. \_ \implies \_ \implies ?\text{merge } ss \ x = ?\text{if } ss \ x$ " is " $\bigwedge x. \_ \implies \_ \implies ?P \ ss \ x$ ")
  <proof>

```

```

lemma (in lbv) merge_not_top_s:
  assumes x: "x  $\in$  A" and ss: "snd' set ss  $\subseteq$  A"
  assumes m: "merge c pc ss x  $\neq$   $\top$ "
  shows "merge c pc ss x = (map snd [(p', t')  $\leftarrow$  ss. p'=pc+1] ++_f x)"
  <proof>

```

### 4.10.3 wtl-inst-list

```

lemmas [iff] = not_Err_eq

```

```

lemma (in lbv) wtl_Nil [simp]: "wtl [] c pc s = s"
  <proof>

```

```

lemma (in lbv) wtl_Cons [simp]:
  "wtl (i#is) c pc s =
  (let s' = wtc c pc s in if s' =  $\top \vee s = \top$  then  $\top$  else wtl is c (pc+1) s')"
  <proof>

```

```

lemma (in lbv) wtl_Cons_not_top:
  "wtl (i#is) c pc s  $\neq$   $\top$  =
  (wtc c pc s  $\neq$   $\top \wedge s \neq \top \wedge$  wtl is c (pc+1) (wtc c pc s)  $\neq$   $\top$ )"
  <proof>

```

```

lemma (in lbv) wtl_top [simp]: "wtl ls c pc  $\top$  =  $\top$ "
  <proof>

```

```

lemma (in lbv) wtl_not_top:
  "wtl ls c pc s  $\neq$   $\top \implies s \neq \top$ "
  <proof>

```

```

lemma (in lbv) wtl_append [simp]:
  " $\bigwedge pc \ s. \text{wtl } (a@b) \ c \ pc \ s = \text{wtl } b \ c \ (pc+\text{length } a) \ (\text{wtl } a \ c \ pc \ s)$ "
  <proof>

```

```

lemma (in lbv) wtl_take:
  "wtl is c pc s  $\neq$   $\top \implies \text{wtl } (\text{take } pc' \ is) \ c \ pc \ s \neq \top$ "
  (is "?wtl is  $\neq$   $\_ \implies \_$ ")
  <proof>

```

```

lemma take_Suc:
  " $\forall n. n < \text{length } l \longrightarrow \text{take } (\text{Suc } n) \ l = (\text{take } n \ l)@[l!n]$ " (is "?P l")
  <proof>

```

```

lemma (in lbv) wtl_Suc:

```

```

    assumes suc: "pc+1 < length is"
    assumes wtl: "wtl (take pc is) c 0 s ≠ ⊤"
    shows "wtl (take (pc+1) is) c 0 s = wtc c pc (wtl (take pc is) c 0 s)"
  <proof>

```

```

lemma (in lbv) wtl_all:
  assumes all: "wtl is c 0 s ≠ ⊤" (is "?wtl is ≠ _")
  assumes pc: "pc < length is"
  shows "wtc c pc (wtl (take pc is) c 0 s) ≠ ⊤"
  <proof>

```

#### 4.10.4 preserves-type

```

lemma (in lbv) merge_pres:
  assumes s0: "snd' set ss ⊆ A" and x: "x ∈ A"
  shows "merge c pc ss x ∈ A"
  <proof>

```

```

lemma pres_typeD2:
  "pres_type step n A ⇒ s ∈ A ⇒ p < n ⇒ snd' set (step p s) ⊆ A"
  <proof>

```

```

lemma (in lbv) wti_pres [intro?]:
  assumes pres: "pres_type step n A"
  assumes cert: "c!(pc+1) ∈ A"
  assumes s_pc: "s ∈ A" "pc < n"
  shows "wti c pc s ∈ A"
  <proof>

```

```

lemma (in lbv) wtc_pres:
  assumes pres: "pres_type step n A"
  assumes cert: "c!pc ∈ A" and cert': "c!(pc+1) ∈ A"
  assumes s: "s ∈ A" and pc: "pc < n"
  shows "wtc c pc s ∈ A"
  <proof>

```

```

lemma (in lbv) wtl_pres:
  assumes pres: "pres_type step (length is) A"
  assumes cert: "cert_ok c (length is) ⊤ ⊥ A"
  assumes s: "s ∈ A"
  assumes all: "wtl is c 0 s ≠ ⊤"
  shows "pc < length is ⇒ wtl (take pc is) c 0 s ∈ A"
    (is "?len pc ⇒ ?wtl pc ∈ A")
  <proof>

```

end

### 4.11 Correctness of the LBV

theory LBVCorrect

```

imports LBVSpec Typing_Framework
begin

locale lbvs = lbv +
  fixes s0  :: 'a (<s0>)
  fixes c   :: "'a list"
  fixes ins :: "'b list"
  fixes phi :: "'a list" (<φ>)
  defines phi_def:
    "φ ≡ map (λpc. if c!pc = ⊥ then wtl (take pc ins) c 0 s0 else c!pc)
      [0.. $\text{length ins}$ ]"

  assumes bounded: "bounded step (length ins)"
  assumes cert: "cert_ok c (length ins) ⊤ ⊥ A"
  assumes pres: "pres_type step (length ins) A"

lemma (in lbvs) phi_None [intro?]:
  "[ pc < length ins; c!pc = ⊥ ] ⇒ φ ! pc = wtl (take pc ins) c 0 s0"
  <proof>

lemma (in lbvs) phi_Some [intro?]:
  "[ pc < length ins; c!pc ≠ ⊥ ] ⇒ φ ! pc = c ! pc"
  <proof>

lemma (in lbvs) phi_len [simp]:
  "length φ = length ins"
  <proof>

lemma (in lbvs) wtl_suc_pc:
  assumes all: "wtl ins c 0 s0 ≠ ⊤"
  assumes pc: "pc+1 < length ins"
  shows "wtl (take (pc+1) ins) c 0 s0 ⊑r φ!(pc+1)"
  <proof>

lemma (in lbvs) wtl_stable:
  assumes wtl: "wtl ins c 0 s0 ≠ ⊤"
  assumes s0: "s0 ∈ A"
  assumes pc: "pc < length ins"
  shows "stable r step φ pc"
  <proof>

lemma (in lbvs) phi_not_top:
  assumes wtl: "wtl ins c 0 s0 ≠ ⊤"
  assumes pc: "pc < length ins"
  shows "φ!pc ≠ ⊤"
  <proof>

lemma (in lbvs) phi_in_A:
  assumes wtl: "wtl ins c 0 s0 ≠ ⊤"
  assumes s0: "s0 ∈ A"

```

```

  shows " $\varphi \in \text{list } (\text{length ins}) A$ "
  <proof>

```

```

lemma (in lbvs) phi0:
  assumes wtl: "wtl ins c 0 s0  $\neq \top$ "
  assumes 0: "0 < length ins"
  shows "s0  $\leq_r \varphi!0$ "
  <proof>

```

```

theorem (in lbvs) wtl_sound:
  assumes wtl: "wtl ins c 0 s0  $\neq \top$ "
  assumes s0: "s0  $\in A$ "
  shows " $\exists ts. \text{wt\_step } r \top \text{ step } ts$ "
  <proof>

```

```

theorem (in lbvs) wtl_sound_strong:
  assumes wtl: "wtl ins c 0 s0  $\neq \top$ "
  assumes s0: "s0  $\in A$ "
  assumes nz: "0 < length ins"
  shows " $\exists ts \in \text{list } (\text{length ins}) A. \text{wt\_step } r \top \text{ step } ts \wedge s0 \leq_r ts!0$ "
  <proof>

```

```

end

```

## 4.12 Completeness of the LBV

```

theory LBVComplete
imports LBVSpec Typing_Framework
begin

```

```

definition is_target :: "[ 's step_type, 's list, nat ]  $\Rightarrow$  bool" where
  "is_target step phi pc'  $\longleftrightarrow$ 
    ( $\exists pc \ s'. \ pc' \neq pc+1 \wedge pc < \text{length phi} \wedge (pc', s') \in \text{set } (\text{step } pc \ (phi!pc))$ )"

```

```

definition make_cert :: "[ 's step_type, 's list, 's ]  $\Rightarrow$  's certificate" where
  "make_cert step phi B =
    map ( $\lambda pc. \text{if is\_target step phi } pc \text{ then } phi!pc \text{ else } B$ ) [0.. $\text{length phi}$ ] @ [B]"

```

```

lemma [code]:
  "is_target step phi pc' =
    list_ex ( $\lambda pc. \ pc' \neq pc+1 \wedge \text{List.member } (\text{map fst } (\text{step } pc \ (phi!pc))) \ pc'$ ) [0.. $\text{length phi}$ ]"
  <proof>

```

```

locale lbvc = lbv +
  fixes phi :: "'a list" (< $\varphi$ >)
  fixes c   :: "'a list"
  defines cert_def: "c  $\equiv$  make_cert step  $\varphi \perp$ "

```

```

assumes mono: "mono r step (length  $\varphi$ ) A"
assumes pres: "pres_type step (length  $\varphi$ ) A"
assumes phi:  " $\forall pc < \text{length } \varphi. \varphi!pc \in A \wedge \varphi!pc \neq \top$ "
assumes bounded: "bounded step (length  $\varphi$ )"

```

```

assumes B_neq_T: " $\perp \neq \top$ "

```

```

lemma (in lbvc) cert: "cert_ok c (length  $\varphi$ )  $\top \perp A$ "
<proof>

```

```

lemmas [simp del] = split_paired_Ex

```

```

lemma (in lbvc) cert_target [intro?]:
  "[ (pc', s')  $\in$  set (step pc ( $\varphi!pc$ ));
    pc'  $\neq$  pc+1; pc < length  $\varphi$ ; pc' < length  $\varphi$  ]
 $\implies$  c!pc' =  $\varphi!pc'$ "
<proof>

```

```

lemma (in lbvc) cert_approx [intro?]:
  "[ pc < length  $\varphi$ ; c!pc  $\neq \perp$  ]
 $\implies$  c!pc =  $\varphi!pc$ "
<proof>

```

```

lemma (in lbv) le_top [simp, intro]:
  "x  $\leq_r \top$ "
<proof>

```

```

lemma (in lbv) merge_mono:
  assumes less: "ss2  $\leq_r$  ss1"
  assumes x:    "x  $\in A$ "
  assumes ss1:  "snd'set ss1  $\subseteq A$ "
  assumes ss2:  "snd'set ss2  $\subseteq A$ "
  shows "merge c pc ss2 x  $\leq_r$  merge c pc ss1 x" (is "?s2  $\leq_r$  ?s1")
<proof>

```

```

lemma (in lbvc) wti_mono:
  assumes less: "s2  $\leq_r$  s1"
  assumes pc:   "pc < length  $\varphi$ "
  assumes s1:   "s1  $\in A$ "
  assumes s2:   "s2  $\in A$ "
  shows "wti c pc s2  $\leq_r$  wti c pc s1" (is "?s2'  $\leq_r$  ?s1'")
<proof>

```

```

lemma (in lbvc) wtc_mono:
  assumes less: "s2  $\leq_r$  s1"
  assumes pc:   "pc < length  $\varphi$ "
  assumes s1:   "s1  $\in A$ "
  assumes s2:   "s2  $\in A$ "

```

shows "*wtc c pc s2 <=\_r wtc c pc s1*" (is "*?s2' <=\_r ?s1'*")  
*<proof>*

```
lemma (in lbv) top_le_conv [simp]:
  "⊤ ≤r x = (x = ⊤)"
  ⟨proof⟩
```

```
lemma (in lbv) neq_top [simp, elim]:
  "⟦ x ≤r y; y ≠ ⊤ ⟧ ⇒ x ≠ ⊤"
  ⟨proof⟩
```

```

lemma (in lbvc) stable_wti:
  assumes stable: "stable r step  $\varphi$  pc"
  assumes pc:      "pc < length  $\varphi$ "
  shows "wti c pc ( $\varphi$ !pc)  $\neq \top$ "
 $\langle proof \rangle$ 

```

```

lemma (in lbvc) wti_less:
  assumes stable: "stable r step  $\varphi$  pc"
  assumes suc_pc: "Suc pc < length  $\varphi$ "
  shows "wti c pc ( $\varphi$ !pc) <=_r  $\varphi$ !Suc pc" (is "?wti <=_r _")
  <proof>

```

```

lemma (in lbvc) stable_wtc:
  assumes stable: "stable r step phi pc"
  assumes pc:      "pc < length  $\varphi$ "
  shows "wtc c pc ( $\varphi!$ pc)  $\neq \top$ "
 $\langle proof \rangle$ 

```

```

lemma (in lbvc) wtc_less:
  assumes stable: "stable r step  $\varphi$  pc"
  assumes suc_pc: "Suc pc < length  $\varphi$ "
  shows "wtc c pc ( $\varphi!$ pc)  $\leq_r \varphi!$ Suc pc" (is "?wtc  $\leq_r$  _")
 $\langle proof \rangle$ 

```

```

lemma (in lbvc) wt_step_wtl_lemma:
  assumes wt_step: "wt_step r  $\top$  step  $\varphi$ "
  shows " $\bigwedge pc\ s.\ pc + \text{length}\ ls = \text{length}\ \varphi \implies s \leq_r \varphi!pc \implies s \in A \implies s \neq \top \implies$   

 $\text{wtl}\ ls\ c\ pc\ s \neq \top$ "
  (is " $\bigwedge pc\ s.\ \_ \implies \_ \implies \_ \implies \_ \implies ?\text{wtl}\ ls\ pc\ s \neq \_$ ")
  <proof>

```

```

theorem (in lbvc) wtl_complete:
  assumes wt: "wt_step r  $\top$  step  $\varphi$ "
    and s: "s  $\leq_r \varphi!0$ " "s  $\in A$ " "s  $\neq \top$ "
    and len: "length ins = length phi"
  shows "wtl ins c 0 s  $\neq \top$ "
  <proof>

```

end

### 4.13 Abstract Bytecode Verifier

### 4.14 Semilattices

### 4.15 The Java Type System as Semilattice

```

theory JType
imports "../DFA/Semilattices" "../J/WellForm"
begin

definition super :: "'a prog ⇒ cname ⇒ cname" where
  "super G C == fst (the (class G C))"

lemma superI:
  "G ⊢ C <C1 D ⇒⇒ super G C = D"
  ⟨proof⟩

definition is_ref :: "ty ⇒ bool" where
  "is_ref T == case T of PrimT t ⇒ False | RefT r ⇒ True"

definition sup :: "'c prog ⇒ ty ⇒ ty ⇒ ty err" where
  "sup G T1 T2 ==
    case T1 of PrimT P1 ⇒ (case T2 of PrimT P2 ⇒
      (if P1 = P2 then OK (PrimT P1) else Err) | RefT R ⇒ Err)
    | RefT R1 ⇒ (case T2 of PrimT P ⇒ Err | RefT R2 ⇒
      (case R1 of NullT ⇒ (case R2 of NullT ⇒ OK NT | ClassT C ⇒ OK (Class C))
    | ClassT C ⇒ (case R2 of NullT ⇒ OK (Class C)
      | ClassT D ⇒ OK (Class (exec_lub (subcls1 G) (super G) C D))))))"

definition subtype :: "'c prog ⇒ ty ⇒ ty ⇒ bool" where
  "subtype G T1 T2 == G ⊢ T1 ⪯ T2"

definition is_ty :: "'c prog ⇒ ty ⇒ bool" where
  "is_ty G T == case T of PrimT P ⇒ True | RefT R ⇒
    (case R of NullT ⇒ True | ClassT C ⇒ (C, Object) ∈ (subcls1 G)*)"

abbreviation "types G == Collect (is_type G)"

definition esl :: "'c prog ⇒ ty esl" where
  "esl G == (types G, subtype G, sup G)"

lemma PrimT_PrimT: "(G ⊢ xb ⪯ PrimT p) = (xb = PrimT p)"
  ⟨proof⟩

lemma PrimT_PrimT2: "(G ⊢ PrimT p ⪯ xb) = (xb = PrimT p)"
  ⟨proof⟩

lemma is_tyI:
  "[ is_type G T; ws_prog G ] ⇒⇒ is_ty G T"
  ⟨proof⟩

lemma is_type_conv:
  "ws_prog G ⇒⇒ is_type G T = is_ty G T"

```

*<proof>*

**lemma** *order\_widen*:

"acyclic (subcls1 G)  $\implies$  order (subtype G)"  
*<proof>*

**lemma** *wf\_converse\_subcls1\_impl\_acc\_subtype*:

"wf ((subcls1 G)<sup>-1</sup>)  $\implies$  acc (subtype G)"  
*<proof>*

**lemma** *closed\_err\_types*:

"[ ws\_prog G; single\_valued (subcls1 G); acyclic (subcls1 G) ]  
 $\implies$  closed (err (types G)) (lift2 (sup G))"  
*<proof>*

**lemma** *sup\_subtype\_greater*:

"[ ws\_prog G; single\_valued (subcls1 G); acyclic (subcls1 G);  
 is\_type G t1; is\_type G t2; sup G t1 t2 = OK s ]  
 $\implies$  subtype G t1 s  $\wedge$  subtype G t2 s"  
*<proof>*

**lemma** *sup\_subtype\_smallest*:

"[ ws\_prog G; single\_valued (subcls1 G); acyclic (subcls1 G);  
 is\_type G a; is\_type G b; is\_type G c;  
 subtype G a c; subtype G b c; sup G a b = OK d ]  
 $\implies$  subtype G d c"  
*<proof>*

**lemma** *sup\_exists*:

"[ subtype G a c; subtype G b c; sup G a b = Err ]  $\implies$  False"  
*<proof>*

**lemma** *err\_semlat\_JType\_esl\_lemma*:

"[ ws\_prog G; single\_valued (subcls1 G); acyclic (subcls1 G) ]  
 $\implies$  err\_semlat (esl G)"  
*<proof>*

**lemma** *single\_valued\_subcls1*:

"ws\_prog G  $\implies$  single\_valued (subcls1 G)"  
*<proof>*

**theorem** *err\_semlat\_JType\_esl*:

"ws\_prog G  $\implies$  err\_semlat (esl G)"  
*<proof>*

**end**

## 4.16 The JVM Type System as Semilattice

**theory** *JVMType*

**imports** *JType*

**begin**



```

type_synonym locvars_type = "ty err list"
type_synonym opstack_type = "ty list"
type_synonym state_type = "opstack_type × locvars_type"
type_synonym state = "state_type option err"    — for Kildall
type_synonym method_type = "state_type option list" — for BVSpec
type_synonym class_type = "sig ⇒ method_type"
type_synonym prog_type = "cname ⇒ class_type"

definition stk_esl :: "'c prog ⇒ nat ⇒ ty list esl" where
  "stk_esl S maxs == upto_esl maxs (JType.esl S)"

definition reg_sl :: "'c prog ⇒ nat ⇒ ty err list sl" where
  "reg_sl S maxr == Listn.sl maxr (Err.sl (JType.esl S))"

definition sl :: "'c prog ⇒ nat ⇒ nat ⇒ state sl" where
  "sl S maxs maxr ==
    Err.sl (Opt.esl (Product.esl (stk_esl S maxs) (Err.esl (reg_sl S maxr))))"

definition states :: "'c prog ⇒ nat ⇒ nat ⇒ state set" where
  "states S maxs maxr == fst(sl S maxs maxr)"

definition le :: "'c prog ⇒ nat ⇒ nat ⇒ state ord" where
  "le S maxs maxr == fst(snd(sl S maxs maxr))"

definition sup :: "'c prog ⇒ nat ⇒ nat ⇒ state binop" where
  "sup S maxs maxr == snd(snd(sl S maxs maxr))"

definition sup_ty_opt :: "[ 'code prog, ty err, ty err ] ⇒ bool"
  (<_ ⊢ _ <=o _> [71,71] 70) where
  "sup_ty_opt G == Err.le (subtype G)"

definition sup_loc :: "[ 'code prog, locvars_type, locvars_type ] ⇒ bool"
  (<_ ⊢ _ <=l _> [71,71] 70) where
  "sup_loc G == Listn.le (sup_ty_opt G)"

definition sup_state :: "[ 'code prog, state_type, state_type ] ⇒ bool"
  (<_ ⊢ _ <=s _> [71,71] 70) where
  "sup_state G == Product.le (Listn.le (subtype G)) (sup_loc G)"

definition sup_state_opt :: "[ 'code prog, state_type option, state_type option ] ⇒ bool"
  (<_ ⊢ _ <='' _> [71,71] 70) where
  "sup_state_opt G == Opt.le (sup_state G)"

lemma JVM_states_unfold:
  "states S maxs maxr == err(opt((⋃ {list n (types S) | n. n ≤ maxs}) ×
    list maxr (err(types S))))"
  <proof>

lemma JVM_le_unfold:

```

```
"le S m n ==
  Err.le(Opt.le(Product.le(Listn.le(subtype S))(Listn.le(Err.le(subtype S))))))"
⟨proof⟩
```

```
lemma JVM_le_convert:
  "le G m n (OK t1) (OK t2) = G ⊢ t1 <=′ t2"
⟨proof⟩
```

```
lemma JVM_le_Err_conv:
  "le G m n = Err.le (sup_state_opt G)"
⟨proof⟩
```

```
lemma zip_map [rule_format]:
  "∀ a. length a = length b ⟶
  zip (map f a) (map g b) = map (λ(x,y). (f x, g y)) (zip a b)"
⟨proof⟩
```

```
lemma [simp]: "Err.le r (OK a) (OK b) = r a b"
⟨proof⟩
```

```
lemma stk_convert:
  "Listn.le (subtype G) a b = G ⊢ map OK a <=1 map OK b"
⟨proof⟩
```

```
lemma sup_state_conv:
  "(G ⊢ s1 <=s s2) ==
  (G ⊢ map OK (fst s1) <=1 map OK (fst s2)) ∧ (G ⊢ snd s1 <=1 snd s2)"
⟨proof⟩
```

```
lemma subtype_refl [simp]:
  "subtype G t t"
⟨proof⟩
```

```
theorem sup_ty_opt_refl [simp]:
  "G ⊢ t <=o t"
⟨proof⟩
```

```
lemma le_list_refl2 [simp]:
  "(⋀xs. r xs xs) ⟹ Listn.le r xs xs"
⟨proof⟩
```

```
theorem sup_loc_refl [simp]:
  "G ⊢ t <=1 t"
⟨proof⟩
```

```
theorem sup_state_refl [simp]:
  "G ⊢ s <=s s"
⟨proof⟩
```

```
theorem sup_state_opt_refl [simp]:
  "G ⊢ s <=′ s"
⟨proof⟩
```

**theorem** anyConvErr [simp]:

"(G ⊢ Err ≤<sub>o</sub> any) = (any = Err)"  
 ⟨proof⟩

**theorem** OKanyConvOK [simp]:

"(G ⊢ (OK ty') ≤<sub>o</sub> (OK ty)) = (G ⊢ ty' ≤ ty)"  
 ⟨proof⟩

**theorem** sup\_ty\_opt\_OK:

"G ⊢ a ≤<sub>o</sub> (OK b) ⇒ ∃ x. a = OK x"  
 ⟨proof⟩

**lemma** widen\_PrimT\_conv1 [simp]:

"[ G ⊢ S ≤ T; S = PrimT x ] ⇒ T = PrimT x"  
 ⟨proof⟩

**theorem** sup\_PTS\_eq:

"(G ⊢ OK (PrimT p) ≤<sub>o</sub> X) = (X=Err ∨ X = OK (PrimT p))"  
 ⟨proof⟩

**theorem** sup\_loc\_Nil [iff]:

"(G ⊢ [] ≤<sub>l</sub> XT) = (XT=[])"  
 ⟨proof⟩

**theorem** sup\_loc\_Cons [iff]:

"(G ⊢ (Y#YT) ≤<sub>l</sub> XT) = (∃ X XT'. XT=X#XT' ∧ (G ⊢ Y ≤<sub>o</sub> X) ∧ (G ⊢ YT ≤<sub>l</sub> XT'))"  
 ⟨proof⟩

**theorem** sup\_loc\_Cons2:

"(G ⊢ YT ≤<sub>l</sub> (X#XT)) = (∃ Y YT'. YT=Y#YT' ∧ (G ⊢ Y ≤<sub>o</sub> X) ∧ (G ⊢ YT' ≤<sub>l</sub> XT))"  
 ⟨proof⟩

**lemma** sup\_state\_Cons:

"(G ⊢ (x#xt, a) ≤<sub>s</sub> (y#yt, b)) =  
 ((G ⊢ x ≤ y) ∧ (G ⊢ (xt,a) ≤<sub>s</sub> (yt,b)))"  
 ⟨proof⟩

**theorem** sup\_loc\_length:

"G ⊢ a ≤<sub>l</sub> b ⇒ length a = length b"  
 ⟨proof⟩

**theorem** sup\_loc\_nth:

"[ G ⊢ a ≤<sub>l</sub> b; n < length a ] ⇒ G ⊢ (a!n) ≤<sub>o</sub> (b!n)"  
 ⟨proof⟩

**theorem** all\_nth\_sup\_loc:

"∀ b. length a = length b ⇒ (∀ n. n < length a ⇒ (G ⊢ (a!n) ≤<sub>o</sub> (b!n)))  
 ⇒ (G ⊢ a ≤<sub>l</sub> b)" (is "?P a")  
 ⟨proof⟩

**theorem** *sup\_loc\_append*:

"length a = length b  $\implies$   
 $(G \vdash (a @ x) \leq_l (b @ y)) = ((G \vdash a \leq_l b) \wedge (G \vdash x \leq_l y))$ "  
 <proof>

**theorem** *sup\_loc\_rev [simp]*:

" $(G \vdash (\text{rev } a) \leq_l \text{rev } b) = (G \vdash a \leq_l b)$ "  
 <proof>

**theorem** *sup\_loc\_update [rule\_format]*:

" $\forall n y. (G \vdash a \leq_o b) \longrightarrow n < \text{length } y \longrightarrow (G \vdash x \leq_l y) \longrightarrow$   
 $(G \vdash x[n := a] \leq_l y[n := b])$ " (is "?P x")  
 <proof>

**theorem** *sup\_state\_length [simp]*:

" $G \vdash s2 \leq_s s1 \implies$   
 $\text{length } (\text{fst } s2) = \text{length } (\text{fst } s1) \wedge \text{length } (\text{snd } s2) = \text{length } (\text{snd } s1)$ "  
 <proof>

**theorem** *sup\_state\_append\_snd*:

"length a = length b  $\implies$   
 $(G \vdash (i, a @ x) \leq_s (j, b @ y)) = ((G \vdash (i, a) \leq_s (j, b)) \wedge (G \vdash (i, x) \leq_s (j, y)))$ "  
 <proof>

**theorem** *sup\_state\_append\_fst*:

"length a = length b  $\implies$   
 $(G \vdash (a @ x, i) \leq_s (b @ y, j)) = ((G \vdash (a, i) \leq_s (b, j)) \wedge (G \vdash (x, i) \leq_s (y, j)))$ "  
 <proof>

**theorem** *sup\_state\_Cons1*:

" $(G \vdash (x \# xt, a) \leq_s (yt, b)) =$   
 $(\exists y yt'. yt = y \# yt' \wedge (G \vdash x \preceq y) \wedge (G \vdash (xt, a) \leq_s (yt', b)))$ "  
 <proof>

**theorem** *sup\_state\_Cons2*:

" $(G \vdash (xt, a) \leq_s (y \# yt, b)) =$   
 $(\exists x xt'. xt = x \# xt' \wedge (G \vdash x \preceq y) \wedge (G \vdash (xt', a) \leq_s (yt, b)))$ "  
 <proof>

**theorem** *sup\_state\_ignore\_fst*:

" $G \vdash (a, x) \leq_s (b, y) \implies G \vdash (c, x) \leq_s (c, y)$ "  
 <proof>

**theorem** *sup\_state\_rev\_fst*:

" $(G \vdash (\text{rev } a, x) \leq_s (\text{rev } b, y)) = (G \vdash (a, x) \leq_s (b, y))$ "  
 <proof>

**lemma** *sup\_state\_opt\_None\_any [iff]*:

" $(G \vdash \text{None} \leq' \text{any}) = \text{True}$ "  
 <proof>

```

lemma sup_state_opt_any_None [iff]:
  "(G ⊢ any <= ' None) = (any = None)"
  ⟨proof⟩

lemma sup_state_opt_Some_Some [iff]:
  "(G ⊢ (Some a) <= ' (Some b)) = (G ⊢ a <=s b)"
  ⟨proof⟩

lemma sup_state_opt_any_Some [iff]:
  "(G ⊢ (Some a) <= ' any) = (∃ b. any = Some b ∧ G ⊢ a <=s b)"
  ⟨proof⟩

lemma sup_state_opt_Some_any:
  "(G ⊢ any <= ' (Some b)) = (any = None ∨ (∃ a. any = Some a ∧ G ⊢ a <=s b))"
  ⟨proof⟩

theorem sup_ty_opt_trans [trans]:
  "⌊G ⊢ a <=o b; G ⊢ b <=o c⌋ ⇒ G ⊢ a <=o c"
  ⟨proof⟩

theorem sup_loc_trans [trans]:
  "⌊G ⊢ a <=l b; G ⊢ b <=l c⌋ ⇒ G ⊢ a <=l c"
  ⟨proof⟩

theorem sup_state_trans [trans]:
  "⌊G ⊢ a <=s b; G ⊢ b <=s c⌋ ⇒ G ⊢ a <=s c"
  ⟨proof⟩

theorem sup_state_opt_trans [trans]:
  "⌊G ⊢ a <= ' b; G ⊢ b <= ' c⌋ ⇒ G ⊢ a <= ' c"
  ⟨proof⟩

end

```

## 4.17 Effect of Instructions on the State Type

```

theory Effect
imports JVMType "../JVM/JVMExceptions"
begin

type_synonym succ_type = "(p_count × state_type option) list"

```

Program counter of successor instructions:

```

primrec succs :: "instr ⇒ p_count ⇒ p_count list" where
  "succs (Load idx) pc      = [pc+1]"
| "succs (Store idx) pc     = [pc+1]"
| "succs (LitPush v) pc     = [pc+1]"
| "succs (Getfield F C) pc  = [pc+1]"
| "succs (Putfield F C) pc  = [pc+1]"
| "succs (New C) pc         = [pc+1]"
| "succs (Checkcast C) pc   = [pc+1]"

```

```

| "succs Pop pc           = [pc+1]"
| "succs Dup pc          = [pc+1]"
| "succs Dup_x1 pc       = [pc+1]"
| "succs Dup_x2 pc       = [pc+1]"
| "succs Swap pc         = [pc+1]"
| "succs IAdd pc         = [pc+1]"
| "succs (Ifcmpeq b) pc  = [pc+1, nat (int pc + b)]"
| "succs (Goto b) pc     = [nat (int pc + b)]"
| "succs Return pc       = [pc]"
| "succs (Invoke C mn fpTs) pc = [pc+1]"
| "succs Throw pc        = [pc]"

```

Effect of instruction on the state type:

```

fun eff' :: "instr × jvm_prog × state_type ⇒ state_type"
where
  "eff' (Load idx, G, (ST, LT))      = (ok_val (LT ! idx) # ST, LT)" |
  "eff' (Store idx, G, (ts#ST, LT))  = (ST, LT[idx:= OK ts])" |
  "eff' (LitPush v, G, (ST, LT))      = (the (typeof (λv. None) v) # ST, LT)" |
  "eff' (Getfield F C, G, (oT#ST, LT)) = (snd (the (field (G,C) F)) # ST, LT)" |
  "eff' (Putfield F C, G, (vT#oT#ST, LT)) = (ST,LT)" |
  "eff' (New C, G, (ST,LT))           = (Class C # ST, LT)" |
  "eff' (Checkcast C, G, (RefT rt#ST,LT)) = (Class C # ST,LT)" |
  "eff' (Pop, G, (ts#ST,LT))          = (ST,LT)" |
  "eff' (Dup, G, (ts#ST,LT))          = (ts#ts#ST,LT)" |
  "eff' (Dup_x1, G, (ts1#ts2#ST,LT))  = (ts1#ts2#ts1#ST,LT)" |
  "eff' (Dup_x2, G, (ts1#ts2#ts3#ST,LT)) = (ts1#ts2#ts3#ts1#ST,LT)" |
  "eff' (Swap, G, (ts1#ts2#ST,LT))    = (ts2#ts1#ST,LT)" |
  "eff' (IAdd, G, (PrimT Integer#PrimT Integer#ST,LT)) = (PrimT Integer#ST,LT)" |
  "eff' (Ifcmpeq b, G, (ts1#ts2#ST,LT)) = (ST,LT)" |
  "eff' (Goto b, G, s)                 = s" |
  — Return has no successor instruction in the same method
  "eff' (Return, G, s)                 = s" |
  — Throw always terminates abruptly
  "eff' (Throw, G, s)                 = s" |
  "eff' (Invoke C mn fpTs, G, (ST,LT)) = (let ST' = drop (length fpTs) ST
    in (fst (snd (the (method (G,C) (mn,fpTs))))#(tl ST'),LT))"

```

```

primrec match_any :: "jvm_prog ⇒ p_count ⇒ exception_table ⇒ cname list" where
  "match_any G pc [] = []"
| "match_any G pc (e#es) = (let (start_pc, end_pc, handler_pc, catch_type) = e;
    es' = match_any G pc es
  in
    if start_pc <= pc ∧ pc < end_pc then catch_type#es' else es')"
```

```

primrec match :: "jvm_prog ⇒ xcpt ⇒ p_count ⇒ exception_table ⇒ cname list" where
  "match G X pc [] = []"
| "match G X pc (e#es) =
  (if match_exception_entry G (Xcpt X) pc e then [Xcpt X] else match G X pc es)"

```

lemma match\_some\_entry:

```

"match G X pc et = (if ∃ e ∈ set et. match_exception_entry G (Xcpt X) pc e then [Xcpt
```

X] else [])"  
 ⟨proof⟩

**fun**

```
xcpt_names :: "instr × jvm_prog × p_count × exception_table ⇒ cname list"
where
  "xcpt_names (Getfield F C, G, pc, et) = match G NullPointer pc et"
| "xcpt_names (Putfield F C, G, pc, et) = match G NullPointer pc et"
| "xcpt_names (New C, G, pc, et)          = match G OutOfMemory pc et"
| "xcpt_names (Checkcast C, G, pc, et)    = match G ClassCast pc et"
| "xcpt_names (Throw, G, pc, et)          = match_any G pc et"
| "xcpt_names (Invoke C m p, G, pc, et)   = match_any G pc et"
| "xcpt_names (i, G, pc, et)              = []"
```

**definition** xcpt\_eff :: "instr ⇒ jvm\_prog ⇒ p\_count ⇒ state\_type option ⇒ exception\_table ⇒ succ\_type" **where**  
 "xcpt\_eff i G pc s et ==  
 map (λC. (the (match\_exception\_table G C pc et), case s of None ⇒ None | Some s' ⇒  
 Some ([Class C], snd s')))  
 (xcpt\_names (i,G,pc,et))"

**definition** norm\_eff :: "instr ⇒ jvm\_prog ⇒ state\_type option ⇒ state\_type option" **where**  
 "norm\_eff i G == map\_option (λs. eff' (i,G,s))"

**definition** eff :: "instr ⇒ jvm\_prog ⇒ p\_count ⇒ exception\_table ⇒ state\_type option ⇒ succ\_type" **where**  
 "eff i G pc et s == (map (λpc'. (pc', norm\_eff i G s)) (succs i pc)) @ (xcpt\_eff i G pc s et)"

**definition** isPrimT :: "ty ⇒ bool" **where**  
 "isPrimT T == case T of PrimT T' ⇒ True | RefT T' ⇒ False"

**definition** isRefT :: "ty ⇒ bool" **where**  
 "isRefT T == case T of PrimT T' ⇒ False | RefT T' ⇒ True"

**lemma** isPrimT [simp]:  
 "isPrimT T = (∃ T'. T = PrimT T')" ⟨proof⟩

**lemma** isRefT [simp]:  
 "isRefT T = (∃ T'. T = RefT T')" ⟨proof⟩

**lemma** "list\_all2 P a b ⇒ ∀ (x,y) ∈ set (zip a b). P x y"  
 ⟨proof⟩

Conditions under which eff is applicable:

**fun**

```
app' :: "instr × jvm_prog × p_count × nat × ty × state_type ⇒ bool"
where
  "app' (Load idx, G, pc, maxs, rT, s) =  

  (idx < length (snd s) ∧ (snd s) ! idx ≠ Err ∧ length (fst s) < maxs)" |  

  "app' (Store idx, G, pc, maxs, rT, (ts#ST, LT)) =  

  (idx < length LT)" |
```

```

"app' (LitPush v, G, pc, maxs, rT, s) =
  (length (fst s) < maxs ∧ typeof (λt. None) v ≠ None)" |
"app' (Getfield F C, G, pc, maxs, rT, (oT#ST, LT)) =
  (is_class G C ∧ field (G,C) F ≠ None ∧ fst (the (field (G,C) F)) = C ∧
   G ⊢ oT ≤ (Class C))" |
"app' (Putfield F C, G, pc, maxs, rT, (vT#oT#ST, LT)) =
  (is_class G C ∧ field (G,C) F ≠ None ∧ fst (the (field (G,C) F)) = C ∧
   G ⊢ oT ≤ (Class C) ∧ G ⊢ vT ≤ (snd (the (field (G,C) F))))" |
"app' (New C, G, pc, maxs, rT, s) =
  (is_class G C ∧ length (fst s) < maxs)" |
"app' (Checkcast C, G, pc, maxs, rT, (RefT rt#ST,LT)) =
  (is_class G C)" |
"app' (Pop, G, pc, maxs, rT, (ts#ST,LT)) =
  True" |
"app' (Dup, G, pc, maxs, rT, (ts#ST,LT)) =
  (1+length ST < maxs)" |
"app' (Dup_x1, G, pc, maxs, rT, (ts1#ts2#ST,LT)) =
  (2+length ST < maxs)" |
"app' (Dup_x2, G, pc, maxs, rT, (ts1#ts2#ts3#ST,LT)) =
  (3+length ST < maxs)" |
"app' (Swap, G, pc, maxs, rT, (ts1#ts2#ST,LT)) =
  True" |
"app' (IAdd, G, pc, maxs, rT, (PrimT Integer#PrimT Integer#ST,LT)) =
  True" |
"app' (Ifcmpeq b, G, pc, maxs, rT, (ts#ts'#ST,LT)) =
  (0 ≤ int pc + b ∧ (isPrimT ts ∧ ts' = ts ∨ isRefT ts ∧ isRefT ts'))" |
"app' (Goto b, G, pc, maxs, rT, s) =
  (0 ≤ int pc + b)" |
"app' (Return, G, pc, maxs, rT, (T#ST,LT)) =
  (G ⊢ T ≤ rT)" |
"app' (Throw, G, pc, maxs, rT, (T#ST,LT)) =
  isRefT T" |
"app' (Invoke C mn fpTs, G, pc, maxs, rT, s) =
  (length fpTs < length (fst s) ∧
   (let apTs = rev (take (length fpTs) (fst s));
    X = hd (drop (length fpTs) (fst s))
   in
    G ⊢ X ≤ Class C ∧ is_class G C ∧ method (G,C) (mn,fpTs) ≠ None ∧
    list_all2 (λx y. G ⊢ x ≤ y) apTs fpTs))" |

"app' (i,G, pc,maxs,rT,s) = False"

definition xcpt_app :: "instr ⇒ jvm_prog ⇒ nat ⇒ exception_table ⇒ bool" where
  "xcpt_app i G pc et ≡ ∀ C ∈ set(xcpt_names (i,G,pc,et)). is_class G C"

definition app :: "instr ⇒ jvm_prog ⇒ nat ⇒ ty ⇒ nat ⇒ exception_table ⇒ state_type
option ⇒ bool" where
  "app i G maxs rT pc et s == case s of None ⇒ True | Some t ⇒ app' (i,G,pc,maxs,rT,t)
  ∧ xcpt_app i G pc et"

lemma match_any_match_table:
  "C ∈ set (match_any G pc et) ⇒ match_exception_table G C pc et ≠ None"
  ⟨proof⟩

```



**lemma** *match\_X\_match\_table*:

" $C \in \text{set } (\text{match } G \ X \ pc \ et) \implies \text{match\_exception\_table } G \ C \ pc \ et \neq \text{None}$ "  
 ⟨proof⟩

**lemma** *xcpt\_names\_in\_et*:

" $C \in \text{set } (\text{xcpt\_names } (i, G, pc, et)) \implies$   
 $\exists e \in \text{set } et. \text{the } (\text{match\_exception\_table } G \ C \ pc \ et) = \text{fst } (\text{snd } (\text{snd } e))$ "  
 ⟨proof⟩

**lemma** 1: " $2 < \text{length } a \implies (\exists l \ l' \ l''. \text{ls. } a = l\#l'\#l''\#\text{ls})$ "

⟨proof⟩

**lemma** 2: " $\neg(2 < \text{length } a) \implies a = [] \vee (\exists l. a = [l]) \vee (\exists l \ l'. a = [l, l'])$ "

⟨proof⟩

**lemmas** [simp] = app\_def xcpt\_app\_def

simp rules for app

**lemma** *appNone[simp]*: " $\text{app } i \ G \ \text{maxs } rT \ pc \ et \ \text{None} = \text{True}$ " ⟨proof⟩

**lemma** *appLoad[simp]*:

" $(\text{app } (\text{Load } \text{idx}) \ G \ \text{maxs } rT \ pc \ et \ (\text{Some } s)) = (\exists ST \ LT. s = (ST, LT) \wedge \text{idx} < \text{length } LT \wedge$   
 $LT! \text{idx} \neq \text{Err} \wedge \text{length } ST < \text{maxs})$ "  
 ⟨proof⟩

**lemma** *appStore[simp]*:

" $(\text{app } (\text{Store } \text{idx}) \ G \ \text{maxs } rT \ pc \ et \ (\text{Some } s)) = (\exists ts \ ST \ LT. s = (ts\#ST, LT) \wedge \text{idx} < \text{length } LT)$ "  
 ⟨proof⟩

**lemma** *appLitPush[simp]*:

" $(\text{app } (\text{LitPush } v) \ G \ \text{maxs } rT \ pc \ et \ (\text{Some } s)) = (\exists ST \ LT. s = (ST, LT) \wedge \text{length } ST < \text{maxs}$   
 $\wedge \text{typeof } (\lambda v. \text{None}) \ v \neq \text{None})$ "  
 ⟨proof⟩

**lemma** *appGetField[simp]*:

" $(\text{app } (\text{Getfield } F \ C) \ G \ \text{maxs } rT \ pc \ et \ (\text{Some } s)) =$   
 $(\exists oT \ vT \ ST \ LT. s = (oT\#ST, LT) \wedge \text{is\_class } G \ C \wedge$   
 $\text{field } (G, C) \ F = \text{Some } (C, vT) \wedge G \vdash oT \preceq (\text{Class } C) \wedge (\forall x \in \text{set } (\text{match } G \ \text{NullPointer}$   
 $\text{pc } et). \text{is\_class } G \ x))$ "  
 ⟨proof⟩

**lemma** *appPutField[simp]*:

" $(\text{app } (\text{Putfield } F \ C) \ G \ \text{maxs } rT \ pc \ et \ (\text{Some } s)) =$   
 $(\exists vT \ vT' \ oT \ ST \ LT. s = (vT\#oT\#ST, LT) \wedge \text{is\_class } G \ C \wedge$   
 $\text{field } (G, C) \ F = \text{Some } (C, vT') \wedge G \vdash oT \preceq (\text{Class } C) \wedge G \vdash vT \preceq vT' \wedge$   
 $(\forall x \in \text{set } (\text{match } G \ \text{NullPointer } pc \ et). \text{is\_class } G \ x))$ "  
 ⟨proof⟩

**lemma** *appNew[simp]*:

" $(\text{app } (\text{New } C) \ G \ \text{maxs } rT \ pc \ et \ (\text{Some } s)) =$

```

( $\exists$  ST LT.  $s=(ST,LT) \wedge is\_class\ G\ C \wedge length\ ST < maxs \wedge$ 
 $(\forall x \in set\ (match\ G\ OutOfMemory\ pc\ et). is\_class\ G\ x))$ "
<proof>

```

```

lemma appCheckcast[simp]:
  "(app (Checkcast C) G maxs rT pc et (Some s)) =
  ( $\exists rT\ ST\ LT. s = (RefT\ rT\#ST,LT) \wedge is\_class\ G\ C \wedge$ 
   $(\forall x \in set\ (match\ G\ ClassCast\ pc\ et). is\_class\ G\ x))$ "
  <proof>

```

```

lemma appPop[simp]:
  "(app Pop G maxs rT pc et (Some s)) = ( $\exists ts\ ST\ LT. s = (ts\#ST,LT)$ )"
  <proof>

```

```

lemma appDup[simp]:
  "(app Dup G maxs rT pc et (Some s)) = ( $\exists ts\ ST\ LT. s = (ts\#ST,LT) \wedge 1+length\ ST < maxs$ )"
  <proof>

```

```

lemma appDup_x1[simp]:
  "(app Dup_x1 G maxs rT pc et (Some s)) = ( $\exists ts1\ ts2\ ST\ LT. s = (ts1\#ts2\#ST,LT) \wedge 2+length\ ST < maxs$ )"
  <proof>

```

```

lemma appDup_x2[simp]:
  "(app Dup_x2 G maxs rT pc et (Some s)) = ( $\exists ts1\ ts2\ ts3\ ST\ LT. s = (ts1\#ts2\#ts3\#ST,LT) \wedge 3+length\ ST < maxs$ )"
  <proof>

```

```

lemma appSwap[simp]:
  "app Swap G maxs rT pc et (Some s) = ( $\exists ts1\ ts2\ ST\ LT. s = (ts1\#ts2\#ST,LT)$ )"
  <proof>

```

```

lemma appIAdd[simp]:
  "app IAdd G maxs rT pc et (Some s) = ( $\exists\ ST\ LT. s = (PrimT\ Integer\#PrimT\ Integer\#ST,LT)$ )"
  (is "?app s = ?P s")
  <proof>

```

```

lemma appIfcmpeq[simp]:
  "app (Ifcmpeq b) G maxs rT pc et (Some s) =
  ( $\exists ts1\ ts2\ ST\ LT. s = (ts1\#ts2\#ST,LT) \wedge 0 \leq int\ pc + b \wedge$ 
   $(\exists p. ts1 = PrimT\ p \wedge ts2 = PrimT\ p) \vee (\exists r\ r'. ts1 = RefT\ r \wedge ts2 = RefT\ r')$ )"
  <proof>

```

```

lemma appReturn[simp]:
  "app Return G maxs rT pc et (Some s) = ( $\exists T\ ST\ LT. s = (T\#ST,LT) \wedge (G \vdash T \preceq rT)$ )"
  <proof>

```

**lemma** appGoto[simp]:

"app (Goto b) G maxs rT pc et (Some s) = (0 ≤ int pc + b)"  
 ⟨proof⟩

**lemma** appThrow[simp]:

"app Throw G maxs rT pc et (Some s) =  
 (∃ T ST LT r. s=(T#ST,LT) ∧ T = RefT r ∧ (∀ C ∈ set (match\_any G pc et). is\_class G C))"  
 ⟨proof⟩

**lemma** appInvoke[simp]:

"app (Invoke C mn fpTs) G maxs rT pc et (Some s) = (∃ apTs X ST LT mD' rT' b'.  
 s = ((rev apTs) @ (X # ST), LT) ∧ length apTs = length fpTs ∧ is\_class G C ∧  
 G ⊢ X ≤ Class C ∧ (∀ (aT,fT)∈set(zip apTs fpTs). G ⊢ aT ≤ fT) ∧  
 method (G,C) (mn,fpTs) = Some (mD', rT', b') ∧  
 (∀ C ∈ set (match\_any G pc et). is\_class G C))" (is "?app s = ?P s")  
 ⟨proof⟩

**lemma** effNone:

"(pc', s') ∈ set (eff i G pc et None) ⇒ s' = None"  
 ⟨proof⟩

**lemma** xcpt\_app\_lemma [code]:

"xcpt\_app i G pc et = list\_all (is\_class G) (xcpt\_names (i, G, pc, et))"  
 ⟨proof⟩

lemmas [simp del] = app\_def xcpt\_app\_def

end

## 4.18 Monotonicity of eff and app

**theory** EffectMono

**imports** Effect

**begin**

**lemma** PrimT\_PrimT: "(G ⊢ xb ≤ PrimT p) = (xb = PrimT p)"  
 ⟨proof⟩

**lemma** sup\_loc\_some [rule\_format]:

"∀ y n. (G ⊢ b ≤<sub>l</sub> y) ⇒ n < length y ⇒ y!n = OK t ⇒  
 (∃ t. b!n = OK t ∧ (G ⊢ (b!n) ≤<sub>o</sub> (y!n)))"  
 ⟨proof⟩

**lemma** all\_widen\_is\_sup\_loc:

"∀ b. length a = length b ⇒  
 (∀ (x, y) ∈ set (zip a b). G ⊢ x ≤ y) = (G ⊢ (map OK a) ≤<sub>l</sub> (map OK b))"  
 (is "∀ b. length a = length b ⇒ ?Q a b" is "?P a")  
 ⟨proof⟩

```

lemma append_length_n [rule_format]:
  "∀ n. n ≤ length x ⟶ (∃ a b. x = a@b ∧ length a = n)"
⟨proof⟩

lemma rev_append_cons:
  "n < length x ⟹ ∃ a b c. x = (rev a) @ b # c ∧ length a = n"
⟨proof⟩

lemma sup_loc_length_map:
  "G ⊢ map f a <=l map g b ⟹ length a = length b"
⟨proof⟩

lemmas [iff] = not_Err_eq

lemma app_mono:
  "[G ⊢ s <= s'; app i G m rT pc et s'] ⟹ app i G m rT pc et s"
⟨proof⟩

lemmas [simp del] = split_paired_Ex

lemma eff'_mono:
  "[[ app i G m rT pc et (Some s2); G ⊢ s1 <=s s2 ] ⟹
   G ⊢ eff' (i,G,s1) <=s eff' (i,G,s2)]"
⟨proof⟩

lemmas [iff del] = not_Err_eq

end

```

## 4.19 The Bytecode Verifier

```

theory BVSpec
imports Effect
begin

```

This theory contains a specification of the BV. The specification describes correct typings of method bodies; it corresponds to type *checking*.

### definition

— The program counter will always be inside the method:

```

check_bounded :: "instr list ⇒ exception_table ⇒ bool" where
  "check_bounded ins et ⟷
   (∀ pc < length ins. ∀ pc' ∈ set (succs (ins!pc) pc). pc' < length ins) ∧
   (∀ e ∈ set et. fst (snd (snd e)) < length ins)"

```

### definition

— The method type only contains declared classes:

```

check_types :: "jvm_prog ⇒ nat ⇒ nat ⇒ JVMType.state list ⇒ bool" where
  "check_types G mxs mxr phi ⟷ set phi ⊆ states G mxs mxr"

```

### definition

— An instruction is welltyped if it is applicable and its effect  
 — is compatible with the type at all successor instructions:

```

wt_instr :: "[instr,jvm_prog,ty,method_type,nat,p_count,
             exception_table,p_count] ⇒ bool" where
"wt_instr i G rT phi mxs max_pc et pc ⇔
app i G mxs rT pc et (phi!pc) ∧
(∀ (pc',s') ∈ set (eff i G pc et (phi!pc)). pc' < max_pc ∧ G ⊢ s' <= phi!pc)"

```

**definition**

— The type at  $pc=0$  conforms to the method calling convention:

```

wt_start :: "[jvm_prog,cname,ty list,nat,method_type] ⇒ bool" where
"wt_start G C pTs mxl phi ⇔
G ⊢ Some ([],(OK (Class C))#(map OK pTs))@(replicate mxl Err) <= phi!0"

```

**definition**

— A method is welltyped if the body is not empty, if execution does not  
 — leave the body, if the method type covers all instructions and mentions  
 — declared classes only, if the method calling convention is respected, and  
 — if all instructions are welltyped.

```

wt_method :: "[jvm_prog,cname,ty list,ty,nat,nat,instr list,
               exception_table,method_type] ⇒ bool" where
"wt_method G C pTs rT mxs mxl ins et phi ⇔
(let max_pc = length ins in
0 < max_pc ∧
length phi = length ins ∧
check_bounded ins et ∧
check_types G mxs (1+length pTs+mxl) (map OK phi) ∧
wt_start G C pTs mxl phi ∧
(∀ pc. pc < max_pc → wt_instr (ins!pc) G rT phi mxs max_pc et pc))"

```

**definition**

— A program is welltyped if it is wellformed and all methods are welltyped

```

wt_jvm_prog :: "[jvm_prog,prog_type] ⇒ bool" where
"wt_jvm_prog G phi ⇔
wf_prog (λG C (sig,rT,(maxs,maxl,b,et)).
wt_method G C (snd sig) rT maxs maxl b et (phi C sig)) G"

```

**lemma check\_boundedD:**

```

"[[ check_bounded ins et; pc < length ins;
   (pc',s') ∈ set (eff (ins!pc) G pc et s) ]] ⇒
pc' < length ins"
⟨proof⟩

```

**lemma wt\_jvm\_progD:**

```

"wt_jvm_prog G phi ⇒ (∃ wt. wf_prog wt G)"
⟨proof⟩

```

**lemma wt\_jvm\_prog\_impl\_wt\_instr:**

```

"[[ wt_jvm_prog G phi; is_class G C;
   method (G,C) sig = Some (C,rT,maxs,maxl,ins,et); pc < length ins ]]
⇒ wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc"
⟨proof⟩

```

We could leave out the check  $pc' < max\_pc$  in the definition of  $wt\_instr$  in the context of  $wt\_method$ .

**lemma** *wt\_instr\_def2*:

```
"[[ wt_jvm_prog G Phi; is_class G C;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et); pc < length ins;
  i = ins!pc; phi = Phi C sig; max_pc = length ins ]]
⇒ wt_instr i G rT phi maxs max_pc et pc =
  (app i G maxs rT pc et (phi!pc) ∧
   (∀ (pc',s') ∈ set (eff i G pc et (phi!pc)). G ⊢ s' <= ' phi!pc'))"
⟨proof⟩
```

**lemma** *wt\_jvm\_prog\_impl\_wt\_start*:

```
"[[ wt_jvm_prog G phi; is_class G C;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et) ]] ⇒
0 < (length ins) ∧ wt_start G C (snd sig) maxl (phi C sig)"
⟨proof⟩
```

**end**

## 4.20 The Typing Framework for the JVM

**theory** *Typing\_Framework\_JVM*

**imports** *"../DFA/Abstract\_BV"* *JVMType* *EffectMono* *BVSpec*

**begin**

**definition** *exec* :: *"jvm\_prog ⇒ nat ⇒ ty ⇒ exception\_table ⇒ instr list ⇒ JVMType.state step\_type"* where

```
"exec G maxs rT et bs ==
err_step (size bs) (λpc. app (bs!pc) G maxs rT pc et) (λpc. eff (bs!pc) G pc et)"
```

**definition** *opt\_states* :: *"'c prog ⇒ nat ⇒ nat ⇒ (ty list × ty err list) option set"*

where

```
"opt_states G maxs maxr ≡ opt (⋃ {list n (types G) | n. n ≤ maxs} × list maxr (err (types G)))"
```

### 4.20.1 Executability of *check\_bounded*

**primrec** *list\_all'\_rec* :: *"('a ⇒ nat ⇒ bool) ⇒ nat ⇒ 'a list ⇒ bool"*

where

```
"list_all'_rec P n [] = True"
| "list_all'_rec P n (x#xs) = (P x n ∧ list_all'_rec P (Suc n) xs)"
```

**definition** *list\_all'* :: *"('a ⇒ nat ⇒ bool) ⇒ 'a list ⇒ bool"* where

```
"list_all' P xs ≡ list_all'_rec P 0 xs"
```

**lemma** *list\_all'\_rec*:

```
"list_all'_rec P n xs = (∀ p < size xs. P (xs!p) (p+n))"
⟨proof⟩
```

**lemma** *list\_all'* [iff]:

```
"list_all' P xs = (∀ n < size xs. P (xs!n) n)"
⟨proof⟩
```

**lemma** [code]:

```
"check_bounded ins et =
(list_all' (λi pc. list_all (λpc'. pc' < length ins) (succs i pc)) ins ∧
```

```
list_all (λe. fst (snd (snd e)) < length ins) et)"
⟨proof⟩
```

#### 4.20.2 Connecting JVM and Framework

```
lemma check_bounded_is_bounded:
  "check_bounded ins et ⇒ bounded (λpc. eff (ins!pc) G pc et) (length ins)"
⟨proof⟩
```

```
lemma special_ex_swap_lemma [iff]:
  "(∃ X. (∃ n. X = A n ∧ P n) & Q X) = (∃ n. Q(A n) ∧ P n)"
⟨proof⟩
```

```
lemmas [iff del] = not_None_eq
```

```
theorem exec_pres_type:
  "wf_prog wf_mb S ⇒
  pres_type (exec S maxs rT et bs) (size bs) (states S maxs maxr)"
⟨proof⟩
```

```
lemmas [iff] = not_None_eq
```

```
lemma sup_state_opt_unfold:
  "sup_state_opt G ≡ Opt.le (Product.le (Listn.le (subtype G)) (Listn.le (Err.le (subtype G))))"
⟨proof⟩
```

```
lemma app_mono:
  "app_mono (sup_state_opt G) (λpc. app (bs!pc) G maxs rT pc et) (length bs) (opt_states
  G maxs maxr)"
⟨proof⟩
```

```
lemma list_appendI:
  "⌈a ∈ list x A; b ∈ list y A⌋ ⇒ a @ b ∈ list (x+y) A"
⟨proof⟩
```

```
lemma list_map [simp]:
  "(map f xs ∈ list (length xs) A) = (f ' set xs ⊆ A)"
⟨proof⟩
```

```
lemma [iff]:
  "(OK ' A ⊆ err B) = (A ⊆ B)"
⟨proof⟩
```

```
lemma [intro]:
  "x ∈ A ⇒ replicate n x ∈ list n A"
⟨proof⟩
```

```
lemma lesubstep_type_simple:
  "a <=[Product.le (=) r] b ⇒ a ≤ |r| b"
⟨proof⟩
```

**lemma** *eff\_mono*:

" $\llbracket p < \text{length } bs; s \leq_{\text{sup\_state\_opt } G} t; \text{app } (bs!p) \text{ } G \text{ maxs } rT \text{ pc et } t \rrbracket$   
 $\implies \text{eff } (bs!p) \text{ } G \text{ p et } s \leq_{\text{sup\_state\_opt } G} \text{eff } (bs!p) \text{ } G \text{ p et } t$ "  
 <proof>

**lemma** *order\_sup\_state\_opt*:

" $\text{ws\_prog } G \implies \text{order } (\text{sup\_state\_opt } G)$ "  
 <proof>

**theorem** *exec\_mono*:

" $\text{ws\_prog } G \implies \text{bounded } (\text{exec } G \text{ maxs } rT \text{ et } bs) \text{ (size } bs) \implies$   
 $\text{mono } (\text{JVMTType.le } G \text{ maxs maxr}) \text{ (exec } G \text{ maxs } rT \text{ et } bs) \text{ (size } bs) \text{ (states } G \text{ maxs maxr)}$ "  
 <proof>

**theorem** *semilat\_JVM\_slI*:

" $\text{ws\_prog } G \implies \text{semilat } (\text{JVMTType.sl } G \text{ maxs maxr})$ "  
 <proof>

**lemma** *sl\_triple\_conv*:

" $\text{JVMTType.sl } G \text{ maxs maxr} ==$   
 $(\text{states } G \text{ maxs maxr}, \text{JVMTType.le } G \text{ maxs maxr}, \text{JVMTType.sup } G \text{ maxs maxr})$ "  
 <proof>

**lemma** *is\_type\_pTs*:

" $\llbracket \text{wf\_prog wf\_mb } G; (C, S, fs, mdecls) \in \text{set } G; ((mn, pTs), rT, code) \in \text{set } mdecls \rrbracket$   
 $\implies \text{set } pTs \subseteq \text{types } G$ "  
 <proof>

**lemma** *jvm\_prog\_lift*:

**assumes** *wf*:  
 " $\text{wf\_prog } (\lambda G \text{ } C \text{ } bd. P \text{ } G \text{ } C \text{ } bd) \text{ } G$ "

**assumes** *rule*:

" $\bigwedge \text{wf\_mb } C \text{ } mn \text{ } pTs \text{ } C \text{ } rT \text{ maxs maxl } b \text{ et } bd.$   
 $\text{wf\_prog wf\_mb } G \implies$   
 $\text{method } (G, C) \text{ } (mn, pTs) = \text{Some } (C, rT, maxs, maxl, b, et) \implies$   
 $\text{is\_class } G \text{ } C \implies$   
 $\text{set } pTs \subseteq \text{types } G \implies$   
 $bd = ((mn, pTs), rT, maxs, maxl, b, et) \implies$   
 $P \text{ } G \text{ } C \text{ } bd \implies$   
 $Q \text{ } G \text{ } C \text{ } bd$ "

**shows**

" $\text{wf\_prog } (\lambda G \text{ } C \text{ } bd. Q \text{ } G \text{ } C \text{ } bd) \text{ } G$ "  
 <proof>

**end**



## 4.21 LBV for the JVM

```

theory LBJVM
imports Typing_Framework_JVM
begin

type_synonym prog_cert = "cname  $\Rightarrow$  sig  $\Rightarrow$  JVMType.state list"

definition check_cert :: "jvm_prog  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$  JVMType.state list  $\Rightarrow$  bool"
where
  "check_cert G mxs mxr n cert  $\equiv$  check_types G mxs mxr cert  $\wedge$  length cert = n+1  $\wedge$ 
    ( $\forall i < n$ . cert!i  $\neq$  Err)  $\wedge$  cert!n = OK None"

definition lbvjvm :: "jvm_prog  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$  ty  $\Rightarrow$  exception_table  $\Rightarrow$ 
  JVMType.state list  $\Rightarrow$  instr list  $\Rightarrow$  JVMType.state  $\Rightarrow$  JVMType.state" where
  "lbvjvm G maxs maxr rT et cert bs  $\equiv$ 
    wtl_inst_list bs cert (JVMType.sup G maxs maxr) (JVMType.le G maxs maxr) Err (OK None)
    (exec G maxs rT et bs) 0"

definition wt_lbv :: "jvm_prog  $\Rightarrow$  cname  $\Rightarrow$  ty list  $\Rightarrow$  ty  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$ 
  exception_table  $\Rightarrow$  JVMType.state list  $\Rightarrow$  instr list  $\Rightarrow$  bool" where
  "wt_lbv G C pTs rT mxs mxl et cert ins  $\equiv$ 
    check_bounded ins et  $\wedge$ 
    check_cert G mxs (1+size pTs+mxl) (length ins) cert  $\wedge$ 
    0 < size ins  $\wedge$ 
    (let start = Some ([], (OK (Class C))#((map OK pTs))@(replicate mxl Err));
      result = lbvjvm G mxs (1+size pTs+mxl) rT et cert ins (OK start)
    in result  $\neq$  Err)"

definition wt_jvm_prog_lbv :: "jvm_prog  $\Rightarrow$  prog_cert  $\Rightarrow$  bool" where
  "wt_jvm_prog_lbv G cert  $\equiv$ 
    wf_prog ( $\lambda G C$  (sig, rT, (maxs, maxl, b, et)). wt_lbv G C (snd sig) rT maxs maxl et (cert
    C sig) b) G"

definition mk_cert :: "jvm_prog  $\Rightarrow$  nat  $\Rightarrow$  ty  $\Rightarrow$  exception_table  $\Rightarrow$  instr list
 $\Rightarrow$  method_type  $\Rightarrow$  JVMType.state list" where
  "mk_cert G maxs rT et bs phi  $\equiv$  make_cert (exec G maxs rT et bs) (map OK phi) (OK None)"

definition prg_cert :: "jvm_prog  $\Rightarrow$  prog_type  $\Rightarrow$  prog_cert" where
  "prg_cert G phi C sig  $\equiv$  let (C, rT, (maxs, maxl, ins, et)) = the (method (G, C) sig) in
    mk_cert G maxs rT et ins (phi C sig)"

lemma wt_method_def2:
  fixes pTs and mxl and G and mxs and rT and et and bs and phi
  defines [simp]: "mxr  $\equiv$  1 + length pTs + mxl"
  defines [simp]: "r  $\equiv$  sup_state_opt G"
  defines [simp]: "app0  $\equiv$   $\lambda pc$ . app (bs!pc) G mxs rT pc et"
  defines [simp]: "step0  $\equiv$   $\lambda pc$ . eff (bs!pc) G pc et"

shows
  "wt_method G C pTs rT mxs mxl bs et phi =
    (bs  $\neq$  []  $\wedge$ 
    length phi = length bs  $\wedge$ "

```

```

    check_bounded bs et ∧
    check_types G mxs mxr (map OK phi) ∧
    wt_start G C pTs mxl phi ∧
    wt_app_eff r app0 step0 phi)"
  ⟨proof⟩

```

**lemma check\_certD:**

```

  "check_cert G mxs mxr n cert ⇒ cert_ok cert n Err (OK None) (states G mxs mxr)"
  ⟨proof⟩

```

**lemma wt\_lbv\_wt\_step:**

```

  assumes wf: "wf_prog wf_mb G"
  assumes lbv: "wt_lbv G C pTs rT mxs mxl et cert ins"
  assumes C: "is_class G C"
  assumes pTs: "set pTs ⊆ types G"

```

```

  defines [simp]: "mxr ≡ 1+length pTs+mxl"

```

```

  shows "∃ ts ∈ list (size ins) (states G mxs mxr).

```

```

    wt_step (JVMType.le G mxs mxr) Err (exec G mxs rT et ins) ts
    ∧ OK (Some ([], (OK (Class C))#((map OK pTs))@(replicate mxl Err))) ≤_(JVMType.le
G mxs mxr) ts!0"
  ⟨proof⟩

```

**lemma wt\_lbv\_wt\_method:**

```

  assumes wf: "wf_prog wf_mb G"
  assumes lbv: "wt_lbv G C pTs rT mxs mxl et cert ins"
  assumes C: "is_class G C"
  assumes pTs: "set pTs ⊆ types G"

```

```

  shows "∃ phi. wt_method G C pTs rT mxs mxl ins et phi"

```

⟨proof⟩

**lemma wt\_method\_wt\_lbv:**

```

  assumes wf: "wf_prog wf_mb G"
  assumes wt: "wt_method G C pTs rT mxs mxl ins et phi"
  assumes C: "is_class G C"
  assumes pTs: "set pTs ⊆ types G"

```

```

  defines [simp]: "cert ≡ mk_cert G mxs rT et ins phi"

```

```

  shows "wt_lbv G C pTs rT mxs mxl et cert ins"

```

⟨proof⟩

**theorem jvm\_lbv\_correct:**

```

  "wt_jvm_prog_lbv G Cert ⇒ ∃ Phi. wt_jvm_prog G Phi"

```

⟨proof⟩

**theorem jvm\_lbv\_complete:**

```

"wt_jvm_prog G Phi  $\impl$  wt_jvm_prog_lbv G (prg_cert G Phi)"
<proof>

end

```

## 4.22 BV Type Safety Invariant

```

theory Correct
imports BVSPEC "../JVM/JVMExec"
begin

definition approx_val :: "[jvm_prog, aheap, val, ty err]  $\Rightarrow$  bool" where
  "approx_val G h v any == case any of Err  $\Rightarrow$  True | OK T  $\Rightarrow$  G, h  $\vdash$  v ::  $\leq$  T"

definition approx_loc :: "[jvm_prog, aheap, val list, locvars_type]  $\Rightarrow$  bool" where
  "approx_loc G hp loc LT == list_all2 (approx_val G hp) loc LT"

definition approx_stk :: "[jvm_prog, aheap, opstack, opstack_type]  $\Rightarrow$  bool" where
  "approx_stk G hp stk ST == approx_loc G hp stk (map OK ST)"

definition correct_frame :: "[jvm_prog, aheap, state_type, nat, bytecode]  $\Rightarrow$  frame  $\Rightarrow$  bool"
where
  "correct_frame G hp ==  $\lambda$ (ST, LT) maxl ins (stk, loc, C, sig, pc).
    approx_stk G hp stk ST  $\wedge$  approx_loc G hp loc LT  $\wedge$ 
    pc < length ins  $\wedge$  length loc = length (snd sig) + maxl + 1"

primrec correct_frames :: "[jvm_prog, aheap, prog_type, ty, sig, frame list]  $\Rightarrow$  bool" where
  "correct_frames G hp phi rT0 sig0 [] = True"
| "correct_frames G hp phi rT0 sig0 (f#frs) =
  (let (stk, loc, C, sig, pc) = f in
   ( $\exists$  ST LT rT maxs maxl ins et.
    phi C sig ! pc = Some (ST, LT)  $\wedge$  is_class G C  $\wedge$ 
    method (G, C) sig = Some (C, rT, (maxs, maxl, ins, et))  $\wedge$ 
    ( $\exists$  C' mn pTs. ins ! pc = (Invoke C' mn pTs)  $\wedge$ 
     (mn, pTs) = sig0  $\wedge$ 
     ( $\exists$  apTs D ST' LT'.
      (phi C sig) ! pc = Some ((rev apTs) @ (Class D) # ST', LT')  $\wedge$ 
      length apTs = length pTs  $\wedge$ 
      ( $\exists$  D' rT' maxs' maxl' ins' et'.
       method (G, D) sig0 = Some (D', rT', (maxs', maxl', ins', et'))  $\wedge$ 
       G  $\vdash$  rT0  $\leq$  rT')  $\wedge$ 
      correct_frame G hp (ST, LT) maxl ins f  $\wedge$ 
      correct_frames G hp phi rT sig frs))))"

definition correct_state :: "[jvm_prog, prog_type, jvm_state]  $\Rightarrow$  bool"
  (<_, _  $\vdash$  JVM _  $\sqrt{\phantom{x}}$  [51, 51] 50) where
  "correct_state G phi ==  $\lambda$ (xp, hp, frs).
    case xp of
    None  $\Rightarrow$  (case frs of
      []  $\Rightarrow$  True
    | (f#fs)  $\Rightarrow$  G  $\vdash$  h hp  $\sqrt{\phantom{x}}$   $\wedge$  preallocated hp  $\wedge$ 
      (let (stk, loc, C, sig, pc) = f
        in

```

```

       $\exists rT \text{ maxs maxl ins et s.}$ 
      is_class G C  $\wedge$ 
      method (G,C) sig = Some(C,rT,(maxs,maxl,ins,et))  $\wedge$ 
      phi C sig ! pc = Some s  $\wedge$ 
      correct_frame G hp s maxl ins f  $\wedge$ 
      correct_frames G hp phi rT sig fs))
    / Some x  $\Rightarrow$  frs = []"

```

```

lemma sup_ty_opt_OK:
  "(G  $\vdash$  X  $\leq_o$  (OK T')) = ( $\exists T. X = OK T \wedge G \vdash T \preceq T'$ )"
  <proof>

```

#### 4.22.1 approx-val

```

lemma approx_val_Err [simp,intro!]:
  "approx_val G hp x Err"
  <proof>

```

```

lemma approx_val_OK [iff]:
  "approx_val G hp x (OK T) = (G, hp  $\vdash$  x  $::\preceq$  T)"
  <proof>

```

```

lemma approx_val_Null [simp,intro!]:
  "approx_val G hp Null (OK (RefT x))"
  <proof>

```

```

lemma approx_val_sup_heap:
  "[[ approx_val G hp v T; hp  $\leq_l$  hp' ]  $\Rightarrow$  approx_val G hp' v T"
  <proof>

```

```

lemma approx_val_heap_update:
  "[[ hp a = Some obj'; G, hp  $\vdash$  v  $::\preceq$  T; obj_ty obj = obj_ty obj' ]
 $\Rightarrow$  G, hp(a  $\mapsto$  obj)  $\vdash$  v  $::\preceq$  T"
  <proof>

```

```

lemma approx_val_widen:
  "[[ approx_val G hp v T; G  $\vdash$  T  $\leq_o$  T'; wf_prog wt G ]
 $\Rightarrow$  approx_val G hp v T'"
  <proof>

```

#### 4.22.2 approx-loc

```

lemma approx_loc_Nil [simp,intro!]:
  "approx_loc G hp [] []"
  <proof>

```

```

lemma approx_loc_Cons [iff]:
  "approx_loc G hp (l#ls) (L#LT) =
  (approx_val G hp l L  $\wedge$  approx_loc G hp ls LT)"
  <proof>

```

```

lemma approx_loc_nth:
  "[[ approx_loc G hp loc LT; n < length LT ]

```

$\Rightarrow \text{approx\_val } G \text{ hp } (\text{loc!}n) (LT!n)"$   
 $\langle \text{proof} \rangle$

**lemma** `approx_loc_imp_approx_val_sup`:  
 $"[\text{approx\_loc } G \text{ hp loc } LT; n < \text{length } LT; LT ! n = \text{OK } T; G \vdash T \preceq T'; \text{wf\_prog wt } G]$   
 $\Rightarrow G, \text{hp} \vdash (\text{loc!}n) :: \preceq T'"$   
 $\langle \text{proof} \rangle$

**lemma** `approx_loc_conv_all_nth`:  
 $"\text{approx\_loc } G \text{ hp loc } LT =$   
 $(\text{length loc} = \text{length } LT \wedge (\forall n < \text{length loc}. \text{approx\_val } G \text{ hp } (\text{loc!}n) (LT!n)))"$   
 $\langle \text{proof} \rangle$

**lemma** `approx_loc_sup_heap`:  
 $"[\text{approx\_loc } G \text{ hp loc } LT; \text{hp} \leq / \text{hp}' ]$   
 $\Rightarrow \text{approx\_loc } G \text{ hp}' \text{ loc } LT"$   
 $\langle \text{proof} \rangle$

**lemma** `approx_loc_widen`:  
 $"[\text{approx\_loc } G \text{ hp loc } LT; G \vdash LT \leq_l LT'; \text{wf\_prog wt } G ]$   
 $\Rightarrow \text{approx\_loc } G \text{ hp loc } LT'"$   
 $\langle \text{proof} \rangle$

**lemma** `loc_widen_Err [dest]`:  
 $"\bigwedge XT. G \vdash \text{replicate } n \text{ Err} \leq_l XT \Rightarrow XT = \text{replicate } n \text{ Err}"$   
 $\langle \text{proof} \rangle$

**lemma** `approx_loc_Err [iff]`:  
 $"\text{approx\_loc } G \text{ hp } (\text{replicate } n \text{ v}) (\text{replicate } n \text{ Err})"$   
 $\langle \text{proof} \rangle$

**lemma** `approx_loc_subst`:  
 $"[\text{approx\_loc } G \text{ hp loc } LT; \text{approx\_val } G \text{ hp } x \text{ X } ]$   
 $\Rightarrow \text{approx\_loc } G \text{ hp } (\text{loc}[\text{idx}:=x]) (LT[\text{idx}:=X])"$   
 $\langle \text{proof} \rangle$

**lemma** `approx_loc_append`:  
 $"\text{length } l1 = \text{length } L1 \Rightarrow$   
 $\text{approx\_loc } G \text{ hp } (l1 @ l2) (L1 @ L2) =$   
 $(\text{approx\_loc } G \text{ hp } l1 L1 \wedge \text{approx\_loc } G \text{ hp } l2 L2)"$   
 $\langle \text{proof} \rangle$

### 4.22.3 approx-stk

**lemma** `approx_stk_rev_lem`:  
 $"\text{approx\_stk } G \text{ hp } (\text{rev } s) (\text{rev } t) = \text{approx\_stk } G \text{ hp } s \text{ } t"$   
 $\langle \text{proof} \rangle$

**lemma** `approx_stk_rev`:  
 $"\text{approx\_stk } G \text{ hp } (\text{rev } s) \text{ } t = \text{approx\_stk } G \text{ hp } s (\text{rev } t)"$   
 $\langle \text{proof} \rangle$

**lemma** `approx_stk_sup_heap`:  
 $"[\text{approx\_stk } G \text{ hp stk } ST; \text{hp} \leq / \text{hp}' ] \Rightarrow \text{approx\_stk } G \text{ hp}' \text{ stk } ST"$

*<proof>*

**lemma approx\_stk\_widen:**

" $\llbracket \text{approx\_stk } G \text{ hp stk } ST; G \vdash \text{map OK } ST \leq \text{map OK } ST'; \text{wf\_prog wt } G \rrbracket$   
 $\implies \text{approx\_stk } G \text{ hp stk } ST'$ "

*<proof>*

**lemma approx\_stk\_Nil [iff]:**

" $\text{approx\_stk } G \text{ hp } [] []$ "

*<proof>*

**lemma approx\_stk\_Cons [iff]:**

" $\text{approx\_stk } G \text{ hp } (x\#\text{stk}) (S\#ST) =$   
 $(\text{approx\_val } G \text{ hp } x (\text{OK } S) \wedge \text{approx\_stk } G \text{ hp stk } ST)$ "

*<proof>*

**lemma approx\_stk\_Cons\_lemma [iff]:**

" $\text{approx\_stk } G \text{ hp stk } (S\#ST') =$   
 $(\exists s \text{ stk}'. \text{stk} = s\#\text{stk}' \wedge \text{approx\_val } G \text{ hp } s (\text{OK } S) \wedge \text{approx\_stk } G \text{ hp stk}' ST')$ "

*<proof>*

**lemma approx\_stk\_append:**

" $\text{approx\_stk } G \text{ hp stk } (S@S') \implies$   
 $(\exists s \text{ stk}'. \text{stk} = s@\text{stk}' \wedge \text{length } s = \text{length } S \wedge \text{length } \text{stk}' = \text{length } S' \wedge$   
 $\text{approx\_stk } G \text{ hp } s S \wedge \text{approx\_stk } G \text{ hp stk}' S')$ "

*<proof>*

**lemma approx\_stk\_all\_widen:**

" $\llbracket \text{approx\_stk } G \text{ hp stk } ST; \forall (x, y) \in \text{set } (\text{zip } ST \text{ } ST'). G \vdash x \preceq y; \text{length } ST = \text{length } ST'; \text{wf\_prog wt } G \rrbracket$

$\implies \text{approx\_stk } G \text{ hp stk } ST'$ "

*<proof>*

#### 4.22.4 oconf

**lemma oconf\_field\_update:**

" $\llbracket \text{map\_of } (\text{fields } (G, \text{oT})) \text{ FD} = \text{Some } T; G, \text{hp} \vdash v :: \leq T; G, \text{hp} \vdash (\text{oT}, fs) \checkmark \rrbracket$   
 $\implies G, \text{hp} \vdash (\text{oT}, fs(\text{FD} \mapsto v)) \checkmark$ "

*<proof>*

**lemma oconf\_newref:**

" $\llbracket \text{hp } \text{oref} = \text{None}; G, \text{hp} \vdash \text{obj } \checkmark; G, \text{hp} \vdash \text{obj}' \checkmark \rrbracket \implies G, \text{hp}(\text{oref} \mapsto \text{obj}') \vdash \text{obj } \checkmark$ "

*<proof>*

**lemma oconf\_heap\_update:**

" $\llbracket \text{hp } a = \text{Some } \text{obj}'; \text{obj\_ty } \text{obj}' = \text{obj\_ty } \text{obj}''; G, \text{hp} \vdash \text{obj } \checkmark \rrbracket$   
 $\implies G, \text{hp}(a \mapsto \text{obj}'') \vdash \text{obj } \checkmark$ "

*<proof>*

#### 4.22.5 hconf

**lemma hconf\_newref:**

" $\llbracket \text{hp } \text{oref} = \text{None}; G \vdash \text{hp } \checkmark; G, \text{hp} \vdash \text{obj } \checkmark \rrbracket \implies G \vdash \text{hp}(\text{oref} \mapsto \text{obj}) \checkmark$ "

*<proof>*

```

lemma hconf_field_update:
  "[[ map_of (fields (G, oT)) X = Some T; hp a = Some(oT,fs);
    G, hp ⊢ v :: ≤ T; G ⊢ h hp √ ] ]
  ⇒ G ⊢ h hp (a ↦ (oT, fs(X ↦ v))) √"
  ⟨proof⟩

```

#### 4.22.6 preallocated

```

lemma preallocated_field_update:
  "[[ map_of (fields (G, oT)) X = Some T; hp a = Some(oT,fs);
    G ⊢ h hp √; preallocated hp ] ]
  ⇒ preallocated (hp (a ↦ (oT, fs(X ↦ v))))"
  ⟨proof⟩

```

```

lemma
  assumes none: "hp oref = None" and alloc: "preallocated hp"
  shows preallocated_newref: "preallocated (hp (oref ↦ obj))"
  ⟨proof⟩

```

#### 4.22.7 correct-frames

```

lemmas [simp del] = fun_upd_apply

```

```

lemma correct_frames_field_update [rule_format]:
  "∀ rT C sig.
    correct_frames G hp phi rT sig frs →
    hp a = Some (C, fs) →
    map_of (fields (G, C)) fl = Some fd →
    G, hp ⊢ v :: ≤ fd
    → correct_frames G (hp (a ↦ (C, fs(fl ↦ v)))) phi rT sig frs"
  ⟨proof⟩

```

```

lemma correct_frames_newref [rule_format]:
  "∀ rT C sig.
    hp x = None →
    correct_frames G hp phi rT sig frs →
    correct_frames G (hp (x ↦ obj)) phi rT sig frs"
  ⟨proof⟩

```

```

end

```

### 4.23 BV Type Safety Proof

```

theory BVSpecTypeSafe
imports Correct
begin

```

This theory contains proof that the specification of the bytecode verifier only admits type safe programs.

### 4.23.1 Preliminaries

Simp and intro setup for the type safety proof:

```
lemmas defs1 = sup_state_conv correct_state_def correct_frame_def
           wt_instr_def eff_def norm_eff_def
```

```
lemmas widen_rules[intro] = approx_val_widen approx_loc_widen approx_stk_widen
```

```
lemmas [simp del] = split_paired_All
```

If we have a welltyped program and a conforming state, we can directly infer that the current instruction is well typed:

```
lemma wt_jvm_prog_impl_wt_instr_cor:
  "[[ wt_jvm_prog G phi; method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
    G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ] ]
  ⇒ wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc"
  <proof>
```

### 4.23.2 Exception Handling

Exceptions don't touch anything except the stack:

```
lemma exec_instr_xcpt:
  "(fst (exec_instr i G hp stk vars Cl sig pc frs) = Some xcp)
  = (∃stk'. exec_instr i G hp stk vars Cl sig pc frs =
    (Some xcp, hp, (stk', vars, Cl, sig, pc)#frs))"
  <proof>
```

Relates *match\_any* from the Bytecode Verifier with *match\_exception\_table* from the operational semantics:

```
lemma in_match_any:
  "match_exception_table G xcpt pc et = Some pc' ⇒
  ∃C. C ∈ set (match_any G pc et) ∧ G ⊢ xcpt ≤C C ∧
  match_exception_table G C pc et = Some pc'"
  (is "PROP ?P et" is "?match et ⇒ ?match_any et")
  <proof>
```

```
lemma match_et_imp_match:
  "match_exception_table G (Xcpt X) pc et = Some handler
  ⇒ match G X pc et = [Xcpt X]"
  <proof>
```

We can prove separately that the recursive search for exception handlers (*find\_handler*) in the frame stack results in a conforming state (if there was no matching exception handler in the current frame). We require that the exception is a valid heap address, and that the state before the exception occurred conforms.

```
lemma uncaught_xcpt_correct:
  "∧f. [[ wt_jvm_prog G phi; xcp = Addr adr; hp adr = Some T;
    G,phi ⊢JVM (None, hp, f#frs)√ ] ]
  ⇒ G,phi ⊢JVM (find_handler G (Some xcp) hp frs)√"
  (is "∧f. [[ ?wt; ?adr; ?hp; ?correct (None, hp, f#frs) ] ] ⇒ ?correct (?find frs)")
  <proof>
```



**declare** *raise\_if\_def* [simp]

The requirement of lemma *uncaught\_xcpt\_correct* (that the exception is a valid reference on the heap) is always met for welltyped instructions and conformant states:

```
lemma exec_instr_xcpt_hp:
  "[[ fst (exec_instr (ins!pc) G hp stk vars Cl sig pc frs) = Some xcp;
    wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
    G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ] ]
  ⇒ ∃adr T. xcp = Addr adr ∧ hp adr = Some T"
  (is "[[ ?xcpt; ?wt; ?correct ] ] ⇒ ?thesis")
  <proof>
```

```
lemma cname_of_xcp [intro]:
  "[[ preallocated hp; xcp = Addr (XcptRef x) ] ] ⇒ cname_of hp xcp = Xcpt x"
  <proof>
```

Finally we can state that, whenever an exception occurs, the resulting next state always conforms:

```
lemma xcpt_correct:
  "[[ wt_jvm_prog G phi;
    method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
    wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
    fst (exec_instr (ins!pc) G hp stk loc C sig pc frs) = Some xcp;
    Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs);
    G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ] ]
  ⇒ G,phi ⊢JVM state'√"
  <proof>
```

### 4.23.3 Single Instructions

In this section we look at each single (welltyped) instruction, and prove that the state after execution of the instruction still conforms. Since we have already handled exceptions above, we can now assume, that on exception occurs for this (single step) execution.

**lemmas** [iff] = not\_Err\_eq

```
lemma Load_correct:
  "[[ wf_prog wt G;
    method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
    ins!pc = Load idx;
    wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
    Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs);
    G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ] ]
  ⇒ G,phi ⊢JVM state'√"
  <proof>
```

```
lemma Store_correct:
  "[[ wf_prog wt G;
    method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
    ins!pc = Store idx;
    wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
    Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs);
    G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ] ]
```

$\Rightarrow G, \text{phi} \vdash \text{JVM state}' \checkmark$   
 $\langle \text{proof} \rangle$

**lemma LitPush\_correct:**

"[ wf\_prog wt G;  
 method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);  
 ins!pc = LitPush v;  
 wt\_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;  
 Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs);  
 G,phi  $\vdash$  JVM (None, hp, (stk,loc,C,sig,pc)#frs)  $\checkmark$  ]  
 $\Rightarrow G, \text{phi} \vdash \text{JVM state}' \checkmark$   
 $\langle \text{proof} \rangle$

**lemma Cast\_conf2:**

"[ wf\_prog ok G; G,h  $\vdash$  v ::  $\preceq$  RefT rt; cast\_ok G C h v;  
 G  $\vdash$  Class C  $\preceq$  T; is\_class G C ]  
 $\Rightarrow G, h \vdash v :: \preceq T$   
 $\langle \text{proof} \rangle$

**lemmas** defs2 = defs1 raise\_system\_xcpt\_def

**lemma Checkcast\_correct:**

"[ wt\_jvm\_prog G phi;  
 method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);  
 ins!pc = Checkcast D;  
 wt\_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;  
 Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;  
 G,phi  $\vdash$  JVM (None, hp, (stk,loc,C,sig,pc)#frs)  $\checkmark$ ;  
 fst (exec\_instr (ins!pc) G hp stk loc C sig pc frs) = None ]  
 $\Rightarrow G, \text{phi} \vdash \text{JVM state}' \checkmark$   
 $\langle \text{proof} \rangle$

**lemma Getfield\_correct:**

"[ wt\_jvm\_prog G phi;  
 method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);  
 ins!pc = Getfield F D;  
 wt\_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;  
 Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;  
 G,phi  $\vdash$  JVM (None, hp, (stk,loc,C,sig,pc)#frs)  $\checkmark$ ;  
 fst (exec\_instr (ins!pc) G hp stk loc C sig pc frs) = None ]  
 $\Rightarrow G, \text{phi} \vdash \text{JVM state}' \checkmark$   
 $\langle \text{proof} \rangle$

**lemma Putfield\_correct:**

"[ wf\_prog wt G;  
 method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);  
 ins!pc = Putfield F D;  
 wt\_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;  
 Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;  
 G,phi  $\vdash$  JVM (None, hp, (stk,loc,C,sig,pc)#frs)  $\checkmark$ ;

```

fst (exec_instr (ins!pc) G hp stk loc C sig pc frs) = None ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩

```

**lemma New\_correct:**

```

"[[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins!pc = New X;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√;
  fst (exec_instr (ins!pc) G hp stk loc C sig pc frs) = None ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩

```

**lemmas [simp del] = split\_paired\_Ex**

**lemma Invoke\_correct:**

```

"[[ wt_jvm_prog G phi;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Invoke C' mn pTs;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√;
  fst (exec_instr (ins!pc) G hp stk loc C sig pc frs) = None ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩

```

**lemmas [simp del] = map\_append**

**lemma Return\_correct:**

```

"[[ wt_jvm_prog G phi;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Return;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩

```

**lemmas [simp] = map\_append**

**lemma Goto\_correct:**

```

"[[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Goto branch;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩

```

**lemma** *Ifcmpeq\_correct*:

```
"[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Ifcmpeq branch;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩
```

**lemma** *Pop\_correct*:

```
"[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Pop;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩
```

**lemma** *Dup\_correct*:

```
"[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Dup;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩
```

**lemma** *Dup\_x1\_correct*:

```
"[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Dup_x1;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩
```

**lemma** *Dup\_x2\_correct*:

```
"[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Dup_x2;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ]
⇒ G,phi ⊢JVM state'√"
⟨proof⟩
```

**lemma** *Swap\_correct*:

```
"[ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
```

```

ins ! pc = Swap;
wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√
⇒ G,phi ⊢JVM state'√
⟨proof⟩

```

**lemma IAdd\_correct:**

```

"⌈ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = IAdd;
  wt_instr (ins!pc) G rT (phi C sig) maxs (length ins) et pc;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ⌋
⇒ G,phi ⊢JVM state'√
⟨proof⟩

```

**lemma Throw\_correct:**

```

"⌈ wf_prog wt G;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  ins ! pc = Throw;
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs) ;
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√;
  fst (exec_instr (ins!pc) G hp stk loc C sig pc frs) = None ⌋
⇒ G,phi ⊢JVM state'√
⟨proof⟩

```

The next theorem collects the results of the sections above, i.e. exception handling and the execution step for each instruction. It states type safety for single step execution: in well-typed programs, a conforming state is transformed into another conforming state when one instruction is executed.

**theorem instr\_correct:**

```

"⌈ wt_jvm_prog G phi;
  method (G,C) sig = Some (C,rT,maxs,maxl,ins,et);
  Some state' = exec (G, None, hp, (stk,loc,C,sig,pc)#frs);
  G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√ ⌋
⇒ G,phi ⊢JVM state'√
⟨proof⟩

```

#### 4.23.4 Main

**lemma correct\_state\_impl\_Some\_method:**

```

"G,phi ⊢JVM (None, hp, (stk,loc,C,sig,pc)#frs)√
⇒ ∃meth. method (G,C) sig = Some(C,meth)
⟨proof⟩

```

**lemma BV\_correct\_1 [rule\_format]:**

```

"∧state. ⌈ wt_jvm_prog G phi; G,phi ⊢JVM state√ ⌋
⇒ exec (G,state) = Some state' → G,phi ⊢JVM state'√
⟨proof⟩

```

**lemma L0:**

"[[ xp=None; frs≠[] ] ⇒ (∃ state'. exec (G,xp,hp,frs) = Some state')]"  
 <proof>

**lemma L1:**

"[[wt\_jvm\_prog G phi; G,phi ⊢JVM (xp,hp,frs)√; xp=None; frs≠[]]]  
 ⇒ ∃ state'. exec(G,xp,hp,frs) = Some state' ∧ G,phi ⊢JVM state'√"  
 <proof>

**theorem BV\_correct [rule\_format]:**

"[[ wt\_jvm\_prog G phi; G ⊢ s -jvm→ t ] ⇒ G,phi ⊢JVM s√ → G,phi ⊢JVM t√"  
 <proof>

**theorem BV\_correct\_implies\_approx:**

"[[ wt\_jvm\_prog G phi;  
 G ⊢ s0 -jvm→ (None,hp,(stk,loc,C,sig,pc)#frs); G,phi ⊢JVM s0 √ ]  
 ⇒ approx\_stk G hp stk (fst (the (phi C sig ! pc))) ∧  
 approx\_loc G hp loc (snd (the (phi C sig ! pc)))"  
 <proof>

**lemma**

fixes G :: jvm\_prog (<Γ>)  
 assumes wf: "wf\_prog wf\_mb Γ"  
 shows hconf\_start: "Γ ⊢ h (start\_heap Γ) √"  
 <proof>

**lemma**

fixes G :: jvm\_prog (<Γ>) and Phi :: prog\_type (<Φ>)  
 shows BV\_correct\_initial:  
 "wt\_jvm\_prog Γ Φ ⇒ is\_class Γ C ⇒ method (Γ,C) (m,[]) = Some (C, b)"  
 ⇒ Γ,Φ ⊢JVM start\_state G C m √"  
 <proof>

**theorem typesafe:**

fixes G :: jvm\_prog (<Γ>)  
 and Phi :: prog\_type (<Φ>)  
 assumes welltyped: "wt\_jvm\_prog Γ Φ"  
 and main\_method: "is\_class Γ C" "method (Γ,C) (m,[]) = Some (C, b)"  
 and exec\_all: "G ⊢ start\_state Γ C m -jvm→ s"  
 shows "Γ,Φ ⊢JVM s √"  
 <proof>

**end**

## 4.24 Welltyped Programs produce no Type Errors

**theory BVNoTypeError**

**imports** "../JVM/JVMDefensive" BVSpecTypeSafe

**begin**

Some simple lemmas about the type testing functions of the defensive JVM:

**lemma typeof\_NoneD [simp,dest]:**

```
"typeof (λv. None) v = Some x ⇒ ¬isAddr v"
⟨proof⟩
```

```
lemma isRef_def2:
  "isRef v = (v = Null ∨ (∃ loc. v = Addr loc))"
  ⟨proof⟩
```

```
lemma app'Store[simp]:
  "app' (Store idx, G, pc, maxs, rT, (ST,LT)) = (∃ T ST'. ST = T#ST' ∧ idx < length LT)"
  ⟨proof⟩
```

```
lemma app'GetField[simp]:
  "app' (Getfield F C, G, pc, maxs, rT, (ST,LT)) =
  (∃ oT vT ST'. ST = oT#ST' ∧ is_class G C ∧
  field (G,C) F = Some (C,vT) ∧ G ⊢ oT ≤ Class C)"
  ⟨proof⟩
```

```
lemma app'PutField[simp]:
  "app' (Putfield F C, G, pc, maxs, rT, (ST,LT)) =
  (∃ vT vT' oT ST'. ST = vT#oT#ST' ∧ is_class G C ∧
  field (G,C) F = Some (C, vT') ∧
  G ⊢ oT ≤ Class C ∧ G ⊢ vT ≤ vT')"
  ⟨proof⟩
```

```
lemma app'Checkcast[simp]:
  "app' (Checkcast C, G, pc, maxs, rT, (ST,LT)) =
  (∃ rT ST'. ST = RefT rT#ST' ∧ is_class G C)"
  ⟨proof⟩
```

```
lemma app'Pop[simp]:
  "app' (Pop, G, pc, maxs, rT, (ST,LT)) = (∃ T ST'. ST = T#ST')"
  ⟨proof⟩
```

```
lemma app'Dup[simp]:
  "app' (Dup, G, pc, maxs, rT, (ST,LT)) =
  (∃ T ST'. ST = T#ST' ∧ length ST < maxs)"
  ⟨proof⟩
```

```
lemma app'Dup_x1[simp]:
  "app' (Dup_x1, G, pc, maxs, rT, (ST,LT)) =
  (∃ T1 T2 ST'. ST = T1#T2#ST' ∧ length ST < maxs)"
  ⟨proof⟩
```

```
lemma app'Dup_x2[simp]:
  "app' (Dup_x2, G, pc, maxs, rT, (ST,LT)) =
  (∃ T1 T2 T3 ST'. ST = T1#T2#T3#ST' ∧ length ST < maxs)"
  ⟨proof⟩
```

```
lemma app'Swap[simp]:
```

```
"app' (Swap, G, pc, maxs, rT, (ST,LT)) = ( $\exists$  T1 T2 ST'. ST = T1#T2#ST')"
```

⟨proof⟩

**lemma** app'IAdd[simp]:

```
"app' (IAdd, G, pc, maxs, rT, (ST,LT)) =
( $\exists$  ST'. ST = PrimT Integer#PrimT Integer#ST')"
```

⟨proof⟩

**lemma** app'Ifcmpeq[simp]:

```
"app' (Ifcmpeq b, G, pc, maxs, rT, (ST,LT)) =
( $\exists$  T1 T2 ST'. ST = T1#T2#ST'  $\wedge$  0  $\leq$  b + int pc  $\wedge$ 
( $\exists$  p. T1 = PrimT p  $\wedge$  T1 = T2)  $\vee$ 
( $\exists$  r r'. T1 = RefT r  $\wedge$  T2 = RefT r')))"
```

⟨proof⟩

**lemma** app'Return[simp]:

```
"app' (Return, G, pc, maxs, rT, (ST,LT)) =
( $\exists$  T ST'. ST = T#ST'  $\wedge$  G  $\vdash$  T  $\preceq$  rT)"
```

⟨proof⟩

**lemma** app'Throw[simp]:

```
"app' (Throw, G, pc, maxs, rT, (ST,LT)) =
( $\exists$  ST' r. ST = RefT r#ST')"
```

⟨proof⟩

**lemma** app'Invoke[simp]:

```
"app' (Invoke C mn fpTs, G, pc, maxs, rT, ST, LT) =
( $\exists$  apTs X ST' mD' rT' b'.
ST = (rev apTs) @ X # ST'  $\wedge$ 
length apTs = length fpTs  $\wedge$  is_class G C  $\wedge$ 
( $\forall$  (aT,fT)  $\in$  set(zip apTs fpTs). G  $\vdash$  aT  $\preceq$  fT)  $\wedge$ 
method (G,C) (mn,fpTs) = Some (mD', rT', b')  $\wedge$  G  $\vdash$  X  $\preceq$  Class C)"
(is "?app ST LT = ?P ST LT")
```

⟨proof⟩

**lemma** approx\_loc\_len [simp]:

```
"approx_loc G hp loc LT  $\implies$  length loc = length LT"
```

⟨proof⟩

**lemma** approx\_stk\_len [simp]:

```
"approx_stk G hp stk ST  $\implies$  length stk = length ST"
```

⟨proof⟩

**lemma** isRefI [intro, simp]: "G, hp  $\vdash$  v ::  $\preceq$  RefT T  $\implies$  isRef v"

⟨proof⟩

**lemma** isIntgI [intro, simp]: "G, hp  $\vdash$  v ::  $\preceq$  PrimT Integer  $\implies$  isIntg v"

⟨proof⟩



**lemma list\_all2\_approx:**

"list\_all2 (approx\_val G hp) s (map OK S) = list\_all2 (conf G hp) s S"  
 ⟨proof⟩

**lemma list\_all2\_conf\_widen:**

"wf\_prog mb G  $\implies$   
 list\_all2 (conf G hp) a b  $\implies$   
 list\_all2 ( $\lambda x y. G \vdash x \preceq y$ ) b c  $\implies$   
 list\_all2 (conf G hp) a c"  
 ⟨proof⟩

The main theorem: welltyped programs do not produce type errors if they are started in a conformant state.

**theorem no\_type\_error:**

assumes welltyped: "wt\_jvm\_prog G Phi" and conforms: "G, Phi  $\vdash_{JVM} s \checkmark$ "  
 shows "exec\_d G (Normal s)  $\neq$  TypeError"  
 ⟨proof⟩

The theorem above tells us that, in welltyped programs, the defensive machine reaches the same result as the aggressive one (after arbitrarily many steps).

**theorem welltyped\_aggressive\_imp\_defensive:**

"wt\_jvm\_prog G Phi  $\implies G, \text{Phi} \vdash_{JVM} s \checkmark \implies G \vdash s \text{--jvm} \rightarrow t$   
 $\implies G \vdash (\text{Normal } s) \text{--jvmd} \rightarrow (\text{Normal } t)"$   
 ⟨proof⟩

**lemma neq\_TypeError\_eq [simp]:** "s  $\neq$  TypeError = ( $\exists s'. s = \text{Normal } s'$ )"

⟨proof⟩

**theorem no\_type\_errors:**

"wt\_jvm\_prog G Phi  $\implies G, \text{Phi} \vdash_{JVM} s \checkmark$   
 $\implies G \vdash (\text{Normal } s) \text{--jvmd} \rightarrow t \implies t \neq \text{TypeError}"$   
 ⟨proof⟩

**corollary no\_type\_errors\_initial:**

fixes G ( $\langle \Gamma \rangle$ ) and Phi ( $\langle \Phi \rangle$ )  
 assumes wt: "wt\_jvm\_prog  $\Gamma \Phi$ "  
 assumes is\_class: "is\_class  $\Gamma C$ "  
 and "method": "method ( $\Gamma, C$ ) (m, []) = Some (C, b)"  
 and m: "m  $\neq$  init"  
 defines start: "s  $\equiv$  start\_state  $\Gamma C m$ "

assumes s: " $\Gamma \vdash (\text{Normal } s) \text{--jvmd} \rightarrow t$ "  
 shows "t  $\neq$  TypeError"

⟨proof⟩

As corollary we get that the aggressive and the defensive machine are equivalent for welltyped programs (if started in a conformant state or in the canonical start state)

**corollary welltyped\_commutes:**

fixes G ( $\langle \Gamma \rangle$ ) and Phi ( $\langle \Phi \rangle$ )  
 assumes wt: "wt\_jvm\_prog  $\Gamma \Phi$ " and \*: " $\Gamma, \Phi \vdash_{JVM} s \checkmark$ "  
 shows " $\Gamma \vdash (\text{Normal } s) \text{--jvmd} \rightarrow (\text{Normal } t) = \Gamma \vdash s \text{--jvm} \rightarrow t$ "  
 ⟨proof⟩

```

corollary welltyped_initial_commutates:
  fixes G (<Γ>) and Phi (<Φ>)
  assumes wt: "wt_jvm_prog Γ Φ"
  assumes is_class: "is_class Γ C"
    and "method": "method (Γ,C) (m,[]) = Some (C, b)"
    and m: "m ≠ init"
  defines start: "s ≡ start_state Γ C m"
  shows "Γ ⊢ (Normal s) -jvmd→ (Normal t) = Γ ⊢ s -jvm→ t"
  <proof>

end

```

## 4.25 Kildall for the JVM

```

theory JVM
imports Typing_Framework_JVM
begin

definition kiljvm :: "jvm_prog ⇒ nat ⇒ nat ⇒ ty ⇒ exception_table ⇒
  instr list ⇒ JVMType.state list ⇒ JVMType.state list" where
  "kiljvm G maxs maxr rT et bs ==
  kildall (JVMType.le G maxs maxr) (JVMType.sup G maxs maxr) (exec G maxs rT et bs)"

definition wt_kil :: "jvm_prog ⇒ cname ⇒ ty list ⇒ ty ⇒ nat ⇒ nat ⇒
  exception_table ⇒ instr list ⇒ bool" where
  "wt_kil G C pTs rT mxs mxl et ins ==
  check_bounded ins et ∧ 0 < size ins ∧
  (let first = Some ([],(OK (Class C))#((map OK pTs))@(replicate mxl Err));
    start = OK first#(replicate (size ins - 1) (OK None));
    result = kiljvm G mxs (1+size pTs+mxl) rT et ins start
  in ∀n < size ins. result!n ≠ Err)"

definition wt_jvm_prog_kildall :: "jvm_prog ⇒ bool" where
  "wt_jvm_prog_kildall G ==
  wf_prog (λG C (sig,rT,(maxs,maxl,b,et)). wt_kil G C (snd sig) rT maxs maxl et b) G"

theorem is_bcv_kiljvm:
  "⌈ wf_prog wf_mb G; bounded (exec G maxs rT et bs) (size bs) ⌋ ⇒
  is_bcv (JVMType.le G maxs maxr) Err (exec G maxs rT et bs)
  (size bs) (states G maxs maxr) (kiljvm G maxs maxr rT et bs)"
  <proof>

lemma subset_replicate: "set (replicate n x) ⊆ {x}"
  <proof>

lemma in_set_replicate:
  "x ∈ set (replicate n y) ⇒ x = y"
  <proof>

theorem wt_kil_correct:
  assumes wf: "wf_prog wf_mb G"
  assumes C: "is_class G C"

```

```

assumes pTs: "set pTs  $\subseteq$  types G"

assumes wtk: "wt_kil G C pTs rT maxs mxl et bs"

shows " $\exists$ phi. wt_method G C pTs rT maxs mxl bs et phi"
<proof>

theorem wt_kil_complete:
  assumes wf: "wf_prog wf_mb G"
  assumes C: "is_class G C"
  assumes pTs: "set pTs  $\subseteq$  types G"

  assumes wtm: "wt_method G C pTs rT maxs mxl bs et phi"

  shows "wt_kil G C pTs rT maxs mxl et bs"
<proof>

theorem jvm_kildall_sound_complete:
  "wt_jvm_prog_kildall G = ( $\exists$ Phi. wt_jvm_prog G Phi)"
<proof>

end

```

## 4.26 Example Welltypings

```

theory BVExample
imports
  "../JVM/JVMListExample"
  BVSpecTypeSafe
  JVM
begin

```

This theory shows type correctness of the example program in section 3.6 (p. 56) by explicitly providing a welltyping. It also shows that the start state of the program conforms to the welltyping; hence type safe execution is guaranteed.

### 4.26.1 Setup

Abbreviations for definitions we will have to use often in the proofs below:

```

lemmas name_defs    = list_name_def test_name_def val_name_def next_name_def
lemmas system_defs  = SystemClasses_def ObjectC_def NullPointerC_def
                      OutOfMemoryC_def ClassCastC_def
lemmas class_defs   = list_class_def test_class_def

```

These auxiliary proofs are for efficiency: class lookup, subclass relation, method and field lookup are computed only once:

```

lemma class_Object [simp]:
  "class E Object = Some (undefined, [], [])"
<proof>

```

```
lemma class_NullPointer [simp]:
  "class E (Xcpt NullPointer) = Some (Object, [], [])"
  ⟨proof⟩
```

```
lemma class_OutOfMemory [simp]:
  "class E (Xcpt OutOfMemory) = Some (Object, [], [])"
  ⟨proof⟩
```

```
lemma class_ClassCast [simp]:
  "class E (Xcpt ClassCast) = Some (Object, [], [])"
  ⟨proof⟩
```

```
lemma class_list [simp]:
  "class E list_name = Some list_class"
  ⟨proof⟩
```

```
lemma class_test [simp]:
  "class E test_name = Some test_class"
  ⟨proof⟩
```

```
lemma E_classes [simp]:
  "{C. is_class E C} = {list_name, test_name, Xcpt NullPointer,
                        Xcpt ClassCast, Xcpt OutOfMemory, Object}"
  ⟨proof⟩
```

The subclass relation spelled out:

```
lemma subcls1:
  "subcls1 E = {(list_name, Object), (test_name, Object), (Xcpt NullPointer, Object),
                (Xcpt ClassCast, Object), (Xcpt OutOfMemory, Object)}"
  ⟨proof⟩
```

The subclass relation is acyclic; hence its converse is well founded:

```
lemma notin_rtrancl:
  "(a, b) ∈ r* ⇒ a ≠ b ⇒ (∧y. (a, y) ∉ r) ⇒ False"
  ⟨proof⟩
```

```
lemma acyclic_subcls1_E: "acyclic (subcls1 E)"
  ⟨proof⟩
```

```
lemma wf_subcls1_E: "wf ((subcls1 E)-1)"
  ⟨proof⟩
```

Method and field lookup:

```
lemma method_Object [simp]:
  "method (E, Object) = Map.empty"
  ⟨proof⟩
```

```
lemma method_append [simp]:
  "method (E, list_name) (append_name, [Class list_name]) =
  Some (list_name, PrimT Void, 3, 0, append_ins, [(1, 2, 8, Xcpt NullPointer)])"
  ⟨proof⟩
```

```
lemma method_makelist [simp]:
```

```

"method (E, test_name) (makelist_name, []) =
Some (test_name, PrimT Void, 3, 2, make_list_ins, [])"
⟨proof⟩

lemma field_val [simp]:
  "field (E, list_name) val_name = Some (list_name, PrimT Integer)"
  ⟨proof⟩

lemma field_next [simp]:
  "field (E, list_name) next_name = Some (list_name, Class list_name)"
  ⟨proof⟩

lemma [simp]: "fields (E, Object) = []"
  ⟨proof⟩

lemma [simp]: "fields (E, Xcpt NullPointer) = []"
  ⟨proof⟩

lemma [simp]: "fields (E, Xcpt ClassCast) = []"
  ⟨proof⟩

lemma [simp]: "fields (E, Xcpt OutOfMemory) = []"
  ⟨proof⟩

lemma [simp]: "fields (E, test_name) = []"
  ⟨proof⟩

lemmas [simp] = is_class_def

```

The next definition and three proof rules implement an algorithm to enumerate natural numbers. The command `apply (elim pc_end pc_next pc_0)` transforms a goal of the form

$$pc < n \implies P \ pc$$

into a series of goals

$$P \ 0$$

$$P \ (Suc \ 0)$$

...

$$P \ n$$

```

definition intervall :: "nat  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$  bool" (<_  $\in$  [_, _']>) where
  "x  $\in$  [a, b)  $\equiv$  a  $\leq$  x  $\wedge$  x < b"

```

```

lemma pc_0: "x < n  $\implies$  (x  $\in$  [0, n)  $\implies$  P x)  $\implies$  P x"
  ⟨proof⟩

```

```

lemma pc_next: "x  $\in$  [n0, n)  $\implies$  P n0  $\implies$  (x  $\in$  [Suc n0, n)  $\implies$  P x)  $\implies$  P x"
  ⟨proof⟩

```

```

lemma pc_end: "x  $\in$  [n,n)  $\implies$  P x"
  ⟨proof⟩

```

### 4.26.2 Program structure

The program is structurally wellformed:

```
lemma wf_struct:
  "wf_prog (λG C mb. True) E" (is "wf_prog ?mb E")
  <proof>
```

### 4.26.3 Welltypings

We show welltypings of the methods `append_name` in class `list_name`, and `makelist_name` in class `test_name`:

```
lemmas eff_simps [simp] = eff_def norm_eff_def xcpt_eff_def
declare appInvoke [simp del]
```

```
definition phi_append :: method_type (<φa>) where
  "φa ≡ map (λ(x,y). Some (x, map OK y)) [
    (
      [], [Class list_name, Class list_name]),
    (
      [Class list_name], [Class list_name, Class list_name]),
    (
      [Class list_name], [Class list_name, Class list_name]),
    (
      [Class list_name, Class list_name], [Class list_name, Class list_name]),
    ([NT, Class list_name, Class list_name], [Class list_name, Class list_name]),
    (
      [Class list_name], [Class list_name, Class list_name]),
    (
      [Class list_name, Class list_name], [Class list_name, Class list_name]),
    (
      [PrimT Void], [Class list_name, Class list_name]),
    (
      [Class Object], [Class list_name, Class list_name]),
    (
      [], [Class list_name, Class list_name]),
    (
      [Class list_name], [Class list_name, Class list_name]),
    (
      [Class list_name, Class list_name], [Class list_name, Class list_name]),
    (
      [], [Class list_name, Class list_name]),
    (
      [PrimT Void], [Class list_name, Class list_name])]"
```

```
lemma bounded_append [simp]:
  "check_bounded append_ins [(Suc 0, 2, 8, Xcpt NullPointer)]"
  <proof>
```

```
lemma types_append [simp]: "check_types E 3 (Suc (Suc 0)) (map OK φa)"
  <proof>
```

```
lemma wt_append [simp]:
  "wt_method E list_name [Class list_name] (PrimT Void) 3 0 append_ins
    [(Suc 0, 2, 8, Xcpt NullPointer)] φa"
  <proof>
```

Some abbreviations for readability

```
abbreviation Clist :: ty
  where "Clist == Class list_name"
abbreviation Ctest :: ty
  where "Ctest == Class test_name"
```

```
definition phi_makelist :: method_type (<φm>) where
  "φm ≡ map (λ(x,y). Some (x, y)) [
    (
      [], [OK Ctest, Err      , Err      ]),
```

```

(
  (
    [Clist], [OK Ctest, Err , Err ]),
  (
    [Clist, Clist], [OK Ctest, Err , Err ]),
  (
    [Clist], [OK Clist, Err , Err ]),
  (
    [PrimT Integer, Clist], [OK Clist, Err , Err ]),
  (
    [], [OK Clist, Err , Err ]),
  (
    [Clist], [OK Clist, Err , Err ]),
  (
    [Clist, Clist], [OK Clist, Err , Err ]),
  (
    [Clist], [OK Clist, OK Clist, Err ]),
  (
    [PrimT Integer, Clist], [OK Clist, OK Clist, Err ]),
  (
    [], [OK Clist, OK Clist, Err ]),
  (
    [Clist], [OK Clist, OK Clist, Err ]),
  (
    [Clist, Clist], [OK Clist, OK Clist, Err ]),
  (
    [Clist], [OK Clist, OK Clist, OK Clist]),
  (
    [PrimT Integer, Clist], [OK Clist, OK Clist, OK Clist]),
  (
    [], [OK Clist, OK Clist, OK Clist]),
  (
    [Clist], [OK Clist, OK Clist, OK Clist]),
  (
    [Clist, Clist], [OK Clist, OK Clist, OK Clist]),
  (
    [PrimT Void], [OK Clist, OK Clist, OK Clist]),
  (
    [], [OK Clist, OK Clist, OK Clist]),
  (
    [Clist], [OK Clist, OK Clist, OK Clist]),
  (
    [Clist, Clist], [OK Clist, OK Clist, OK Clist]),
  (
    [PrimT Void], [OK Clist, OK Clist, OK Clist]))]

```

```

lemma bounded_makelist [simp]: "check_bounded make_list_ins []"
  <proof>

```

```

lemma types_makelist [simp]: "check_types E 3 (Suc (Suc (Suc 0))) (map OK  $\varphi_m$ )"
  <proof>

```

```

lemma wt_makelist [simp]:
  "wt_method E test_name [] (PrimT Void) 3 2 make_list_ins []  $\varphi_m$ "
  <proof>

```

The whole program is welltyped:

```

definition Phi :: prog_type (< $\Phi$ >) where
  " $\Phi$  C sg  $\equiv$  if C = test_name  $\wedge$  sg = (makelist_name, []) then  $\varphi_m$  else
    if C = list_name  $\wedge$  sg = (append_name, [Class list_name]) then  $\varphi_a$  else []"

```

```

lemma wf_prog:
  "wt_jvm_prog E  $\Phi$ "
  <proof>

```

#### 4.26.4 Conformance

Execution of the program will be typesafe, because its start state conforms to the welltyping:

```

lemma "E,  $\Phi \vdash_{JVM}$  start_state E test_name makelist_name  $\surd$ "
  <proof>

```

#### 4.26.5 Example for code generation: inferring method types

```

definition test_kil :: "jvm_prog  $\Rightarrow$  cname  $\Rightarrow$  ty list  $\Rightarrow$  ty  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$ 
  exception_table  $\Rightarrow$  instr list  $\Rightarrow$  JVMType.state list" where
  "test_kil G C pTs rT mxs mxl et instr =

```

```

    (let first = Some ([], (OK (Class C))#((map OK pTs))@(replicate mxl Err));
      start = OK first#(replicate (size instr - 1) (OK None))
    in kiljvm G mxs (1+size pTs+mxl) rT et instr start)"

lemma [code]:
  "unstables r step ss =
    fold (λp A. if ¬stable r step ss p then insert p A else A) [0..<size ss] {}"
  ⟨proof⟩

definition some_elem :: "'a set ⇒ 'a" where [code del]:
  "some_elem = (λS. SOME x. x ∈ S)"
code_printing
  constant some_elem ↦ (SML) "(case/ _ of/ Set/ xs/ =>/ hd/ xs)"

This code setup is just a demonstration and not sound!

lemma False
  ⟨proof⟩

lemma [code]:
  "iter f step ss w = while (λ(ss, w). ¬ Set.is_empty w)
    (λ(ss, w).
      let p = some_elem w in propa f (step p (ss ! p)) ss (w - {p}))
    (ss, w)"
  ⟨proof⟩

lemma JVM_sup_unfold [code]:
  "JVMType.sup S m n = lift2 (Opt.sup
    (Product.sup (Listn.sup (JType.sup S))
      (λx y. OK (map2 (lift2 (JType.sup S)) x y))))"
  ⟨proof⟩

lemmas [code] =
  JType.sup_def [unfolded exec_lub_def]
  JVM_le_unfold
  lesub_def
  plussub_def
  wf_class_code
  widen.equation
  match_exception_entry_def

definition test1 where
  "test1 = test_kil E list_name [Class list_name] (PrimT Void) 3 0
    [(Suc 0, 2, 8, Xcpt NullPointerException)] append_ins"
definition test2 where
  "test2 = test_kil E test_name [] (PrimT Void) 3 2 [] make_list_ins"

⟨ML⟩

end

```



```

theory AuxLemmas
imports "../J/JBasis"
begin

```

```

lemma app_nth_greater_len [simp]:
  "length pre ≤ ind ⇒ (pre @ a # post) ! (Suc ind) = (pre @ post) ! ind"
  ⟨proof⟩

```

```

lemma length_takeWhile: "v ∈ set xs ⇒ length (takeWhile (λz. z ≠ v) xs) < length xs"
  ⟨proof⟩

```

```

lemma nth_length_takeWhile [simp]:
  "v ∈ set xs ⇒ xs ! (length (takeWhile (%z. z≠v) xs)) = v"
  ⟨proof⟩

```

```

lemma map_list_update [simp]:
  "⟦ x ∈ set xs; distinct xs ⟧ ⇒
  (map f xs) [length (takeWhile (λz. z ≠ x) xs) := v] = map (f(x:=v)) xs"
  ⟨proof⟩

```

```

lemma split_compose:
  "(case_prod f) ∘ (λ (a,b). ((fa a), (fb b))) = (λ (a,b). (f (fa a) (fb b)))"
  ⟨proof⟩

```

```

lemma split_iter:
  "(λ (a,b,c). ((g1 a), (g2 b), (g3 c))) = (λ (a,p). ((g1 a), (λ (b, c). ((g2 b), (g3 c))) p)))"
  ⟨proof⟩

```

```

lemma singleton_in_set: "A = {a} ⇒ a ∈ A" ⟨proof⟩

```

**lemma** *the\_map\_upd*: "(the  $\circ$  f( $x \mapsto v$ )) = (the  $\circ$  f)( $x := v$ )"  
 <proof>

**lemma** *map\_of\_in\_set*:  
 "(map\_of xs x = None) = (x  $\notin$  set (map fst xs))"  
 <proof>

**lemma** *map\_map\_upd [simp]*:  
 "y  $\notin$  set xs  $\implies$  map (the  $\circ$  f( $y \mapsto v$ )) xs = map (the  $\circ$  f) xs"  
 <proof>

**lemma** *map\_map\_upds [simp]*:  
 "(\forall y \in set ys. y  $\notin$  set xs)  $\implies$  map (the  $\circ$  f(ys[ $\mapsto$ ]vs)) xs = map (the  $\circ$  f) xs"  
 <proof>

**lemma** *map\_upds\_distinct [simp]*:  
 "distinct ys  $\implies$  length ys = length vs  $\implies$  map (the  $\circ$  f(ys[ $\mapsto$ ]vs)) ys = vs"  
 <proof>

**lemma** *map\_of\_map\_as\_map\_upd*:  
 "distinct (map f zs)  $\implies$  map\_of (map ( $\lambda p. (f p, g p)$ ) zs) = Map.empty (map f zs [ $\mapsto$ ]  
 map g zs)"  
 <proof>

**lemma** *map\_upds\_SomeD*:  
 "(m(xs[ $\mapsto$ ]ys)) k = Some y  $\implies$  k  $\in$  (set xs)  $\vee$  (m k = Some y)"  
 <proof>

**lemma** *map\_of\_upds\_SomeD*: "((map\_of m) (xs[ $\mapsto$ ]ys)) k = Some y  
 $\implies$  k  $\in$  (set (xs @ map fst m))"  
 <proof>

**lemma** *map\_of\_map\_prop*:  
 "[map\_of (map f xs) k = Some v;  $\forall x \in$  set xs. P1 x;  $\forall x. P1 x \longrightarrow P2 (f x)$ ]  $\implies$  P2 (k,  
 v)"  
 <proof>

**lemma** *map\_of\_map2*: " $\forall x \in$  set xs. (fst (f x)) = (fst x)  $\implies$   
 map\_of (map f xs) a = map\_option ( $\lambda b. (snd (f (a, b)))$ ) (map\_of xs a)"  
 <proof>

**end**

**theory** *DefsComp*  
**imports** "../JVM/JVMExec"  
**begin**

**definition** *method\_rT* :: "cname  $\times$  ty  $\times$  'c  $\Rightarrow$  ty" **where**  
 "method\_rT mtd == (fst (snd mtd))"

**definition**

$gx :: \text{"xstate"} \Rightarrow \text{val option}$  where  $gx \equiv fst$

**definition**

$gs :: \text{"xstate"} \Rightarrow \text{state}$  where  $gs \equiv snd$

**definition**

$gh :: \text{"xstate"} \Rightarrow \text{aheap}$  where  $gh \equiv fst \circ snd$

**definition**

$gl :: \text{"xstate"} \Rightarrow \text{State.locals}$  where  $gl \equiv snd \circ snd$

**definition**

$gmb :: \text{"'a prog"} \Rightarrow \text{cname} \Rightarrow \text{sig} \Rightarrow \text{'a}$   
 where  $gmb\ G\ cn\ si \equiv snd(snd(the(method\ (G,cn)\ si)))$

**definition**

$gis :: \text{"jvm\_method"} \Rightarrow \text{bytecode}$   
 where  $gis \equiv fst \circ snd \circ snd$

**definition**

$gjmb\_pns :: \text{"java\_mb"} \Rightarrow \text{vname list}$  where  $gjmb\_pns \equiv fst$

**definition**

$gjmb\_lvs :: \text{"java\_mb"} \Rightarrow (\text{vname} \times \text{ty}) \text{list}$  where  $gjmb\_lvs \equiv fst \circ snd$

**definition**

$gjmb\_blk :: \text{"java\_mb"} \Rightarrow \text{stmt}$  where  $gjmb\_blk \equiv fst \circ snd \circ snd$

**definition**

$gjmb\_res :: \text{"java\_mb"} \Rightarrow \text{expr}$  where  $gjmb\_res \equiv snd \circ snd \circ snd$

**definition**

$gjmb\_plns :: \text{"java\_mb"} \Rightarrow \text{vname list}$   
 where  $gjmb\_plns \equiv \lambda jmb. gjmb\_pns\ jmb\ @\ \text{map}\ fst\ (gjmb\_lvs\ jmb)$

**definition**

$glvs :: \text{"java\_mb"} \Rightarrow \text{State.locals} \Rightarrow \text{locvars}$   
 where  $glvs\ jmb\ loc \equiv \text{map}\ (the \circ loc)\ (gjmb\_plns\ jmb)$

lemmas  $gdefs = gx\_def\ gh\_def\ gl\_def\ gmb\_def\ gis\_def\ glvs\_def$

lemmas  $gjmbdefs = gjmb\_pns\_def\ gjmb\_lvs\_def\ gjmb\_blk\_def\ gjmb\_res\_def\ gjmb\_plns\_def$

lemmas  $galldefs = gdefs\ gjmbdefs$

**definition**  $locvars\_locals :: \text{"java\_mb prog"} \Rightarrow \text{cname} \Rightarrow \text{sig} \Rightarrow \text{State.locals} \Rightarrow \text{locvars}$   
 where

$locvars\_locals\ G\ C\ S\ lvs == the\ (lvs\ This)\ \# glvs\ (gmb\ G\ C\ S)\ lvs$

**definition**  $locals\_locvars :: \text{"java\_mb prog"} \Rightarrow \text{cname} \Rightarrow \text{sig} \Rightarrow \text{locvars} \Rightarrow \text{State.locals}$

where

```
"locals_locvars G C S lvs ==
Map.empty ((gjmb_plns (gmb G C S))[↦](tl lvs), This↦(hd lvs))"
```

**definition** *locvars\_xstate* :: "java\_mb prog ⇒ cname ⇒ sig ⇒ xstate ⇒ locvars" where

```
"locvars_xstate G C S xs == locvars_locals G C S (gl xs)"
```

**lemma** *locvars\_xstate\_par\_dep*:

```
"lv1 = lv2 ⇒
locvars_xstate G C S (xcpt1, hp1, lv1) = locvars_xstate G C S (xcpt2, hp2, lv2)"
⟨proof⟩
```

**lemma** *gx\_conv [simp]*: "gx (xcpt, s) = xcpt" ⟨proof⟩

**lemma** *gh\_conv [simp]*: "gh (xcpt, h, l) = h" ⟨proof⟩

end

**theory** *Index*

**imports** *AuxLemmas DefsComp*

**begin**

**definition** *index* :: "java\_mb ⇒ vname ⇒ nat" where

```
"index == λ (pn,lv,blk,res) v.
if v = This
then 0
else Suc (length (takeWhile (λ z. z~=v) (pn @ map fst lv)))"
```

**lemma** *index\_length\_pns*: "

```
[[ i = index (pns,lvars,blk,res) vn;
wf_java_mdecl G C ((mn,pTs),rT, (pns,lvars,blk,res));
vn ∈ set pns]]
⇒ 0 < i ∧ i < Suc (length pns)"
⟨proof⟩
```

**lemma** *index\_length\_lvars*: "

```
[[ i = index (pns,lvars,blk,res) vn;
wf_java_mdecl G C ((mn,pTs),rT, (pns,lvars,blk,res));
vn ∈ set (map fst lvars)]
⇒ (length pns) < i ∧ i < Suc((length pns) + (length lvars))"
```

$\langle proof \rangle$

```
lemma select_at_index :
  "x ∈ set (gjmb_plns (gmb G C S)) ∨ x = This
  ⇒ (the (loc This) # glvs (gmb G C S) loc) ! (index (gmb G C S) x) = the (loc x)"
  ⟨proof⟩
```

```
lemma lift_if: "(f (if b then t else e)) = (if b then (f t) else (f e))"
  ⟨proof⟩
```

```
lemma update_at_index: "
  [ distinct (gjmb_plns (gmb G C S));
  x ∈ set (gjmb_plns (gmb G C S)); x ≠ This ] ⇒
  (locvars_xstate G C S (Norm (h, l))) [index (gmb G C S) x := val] =
  locvars_xstate G C S (Norm (h, l(x ↦ val)))"
  ⟨proof⟩
```

```
lemma index_of_var: "[ xvar ∉ set pns; xvar ∉ set (map fst zs); xvar ≠ This ]
  ⇒ index (pns, zs @ ((xvar, xval) # xys), blk, res) xvar = Suc (length pns + length
  zs)"
  ⟨proof⟩
```

```
definition disjoint_varnames :: "[vname list, (vname × ty) list] ⇒ bool" where
  "disjoint_varnames pns lvars ≡
  distinct pns ∧ unique lvars ∧ This ∉ set pns ∧ This ∉ set (map fst lvars) ∧
  (∀pn∈set pns. pn ∉ set (map fst lvars))"
```

```
lemma index_of_var2: "
  disjoint_varnames pns (lvars_pre @ (vn, ty) # lvars_post)
  ⇒ index (pns, lvars_pre @ (vn, ty) # lvars_post, blk, res) vn =
  Suc (length pns + length lvars_pre)"
  ⟨proof⟩
```

```
lemma wf_java_mdecl_disjoint_varnames:
  "wf_java_mdecl G C (S, rT, (pns, lvars, blk, res))
  ⇒ disjoint_varnames pns lvars"
  ⟨proof⟩
```

```
lemma wf_java_mdecl_length_pTs_pns:
  "wf_java_mdecl G C ((mn, pTs), rT, pns, lvars, blk, res)
  ⇒ length pTs = length pns"
  ⟨proof⟩
```

end

```

theory TranslCompTp
imports Index "../BV/JVMType"
begin

definition comb :: "[ 'a  $\Rightarrow$  'b list  $\times$  'c, 'c  $\Rightarrow$  'b list  $\times$  'd, 'a ]  $\Rightarrow$  'b list  $\times$  'd"
  (infixr <□> 55)
where
  "comb == ( $\lambda$  f1 f2 x0. let (xs1, x1) = f1 x0;
                        (xs2, x2) = f2 x1
                        in (xs1 @ xs2, x2))"

definition comb_nil :: "'a  $\Rightarrow$  'b list  $\times$  'a" where
  "comb_nil a == ([], a)"

lemma comb_nil_left [simp]: "comb_nil □ f = f"
  <proof>

lemma comb_nil_right [simp]: "f □ comb_nil = f"
  <proof>

lemma comb_assoc [simp]: "(fa □ fb) □ fc = fa □ (fb □ fc)"
  <proof>

lemma comb_inv:
  "(xs', x') = (f1 □ f2) x0  $\implies$ 
 $\exists$  xs1 x1 xs2 x2. (xs1, x1) = (f1 x0)  $\wedge$  (xs2, x2) = f2 x1  $\wedge$  xs' = xs1 @ xs2  $\wedge$  x' = x2"
  <proof>

abbreviation (input)
  mt_of :: "method_type  $\times$  state_type  $\Rightarrow$  method_type"
  where "mt_of == fst"

abbreviation (input)
  sttp_of :: "method_type  $\times$  state_type  $\Rightarrow$  state_type"
  where "sttp_of == snd"

definition nochangeST :: "state_type  $\Rightarrow$  method_type  $\times$  state_type" where
  "nochangeST sttp == ([Some sttp], sttp)"

definition pushST :: "[ty list, state_type]  $\Rightarrow$  method_type  $\times$  state_type" where
  "pushST tps == ( $\lambda$  (ST, LT). ([Some (ST, LT)], (tps @ ST, LT)))"

definition dupST :: "state_type  $\Rightarrow$  method_type  $\times$  state_type" where
  "dupST == ( $\lambda$  (ST, LT). ([Some (ST, LT)], (hd ST # ST, LT)))"

definition dup_x1ST :: "state_type  $\Rightarrow$  method_type  $\times$  state_type" where
  "dup_x1ST == ( $\lambda$  (ST, LT). ([Some (ST, LT)],
                                (hd ST # hd (tl ST) # hd ST # (tl (tl ST)), LT)))"

```

**definition** popST :: "[nat, state\_type]  $\Rightarrow$  method\_type  $\times$  state\_type" where  
 "popST n == ( $\lambda$  (ST, LT). ([Some (ST, LT)], (drop n ST, LT)))"

**definition** replST :: "[nat, ty, state\_type]  $\Rightarrow$  method\_type  $\times$  state\_type" where  
 "replST n tp == ( $\lambda$  (ST, LT). ([Some (ST, LT)], (tp # (drop n ST), LT)))"

**definition** storeST :: "[nat, ty, state\_type]  $\Rightarrow$  method\_type  $\times$  state\_type" where  
 "storeST i tp == ( $\lambda$  (ST, LT). ([Some (ST, LT)], (tl ST, LT [i := OK tp])))"

**primrec** compTpExpr :: "java\_mb  $\Rightarrow$  java\_mb prog  $\Rightarrow$  expr  $\Rightarrow$   
 state\_type  $\Rightarrow$  method\_type  $\times$  state\_type"  
**and** compTpExprs :: "java\_mb  $\Rightarrow$  java\_mb prog  $\Rightarrow$  expr list  $\Rightarrow$   
 state\_type  $\Rightarrow$  method\_type  $\times$  state\_type"  
**where**  
 "compTpExpr jmb G (NewC c) = pushST [Class c]"  
 | "compTpExpr jmb G (Cast c e) = (compTpExpr jmb G e)  $\square$  (replST 1 (Class c))"  
 | "compTpExpr jmb G (Lit val) = pushST [the (typeof ( $\lambda$ v. None) val)]"  
 | "compTpExpr jmb G (BinOp bo e1 e2) =  
 (compTpExpr jmb G e1)  $\square$  (compTpExpr jmb G e2)  $\square$   
 (case bo of  
 Eq => popST 2  $\square$  pushST [PrimT Boolean]  $\square$  popST 1  $\square$  pushST [PrimT Boolean]  
 | Add => replST 2 (PrimT Integer))"  
 | "compTpExpr jmb G (LAcc vn) = ( $\lambda$  (ST, LT).  
 pushST [ok\_val (LT ! (index jmb vn))] (ST, LT))"  
 | "compTpExpr jmb G (vn ::= e) =  
 (compTpExpr jmb G e)  $\square$  dupST  $\square$  (popST 1)"  
 | "compTpExpr jmb G ( {cn}e..fn ) =  
 (compTpExpr jmb G e)  $\square$  replST 1 (snd (the (field (G,cn) fn)))"  
 | "compTpExpr jmb G (FAss cn e1 fn e2 ) =  
 (compTpExpr jmb G e1)  $\square$  (compTpExpr jmb G e2)  $\square$  dup\_x1ST  $\square$  (popST 2)"  
 | "compTpExpr jmb G ( {C}a..mn({fpTs}ps) ) =  
 (compTpExpr jmb G a)  $\square$  (compTpExprs jmb G ps)  $\square$   
 (replST ((length ps) + 1) (method\_rT (the (method (G,C) (mn,fpTs)))))"  
 | "compTpExprs jmb G [] = comb\_nil"  
 | "compTpExprs jmb G (e#es) = (compTpExpr jmb G e)  $\square$  (compTpExprs jmb G es)"

**primrec** compTpStmnt :: "java\_mb  $\Rightarrow$  java\_mb prog  $\Rightarrow$  stmt  $\Rightarrow$   
 state\_type  $\Rightarrow$  method\_type  $\times$  state\_type"

**where**  
 "compTpStmnt jmb G Skip = comb\_nil"  
 | "compTpStmnt jmb G (Expr e) = (compTpExpr jmb G e)  $\square$  popST 1"  
 | "compTpStmnt jmb G (c1;; c2) = (compTpStmnt jmb G c1)  $\square$  (compTpStmnt jmb G c2)"  
 | "compTpStmnt jmb G (If(e) c1 Else c2) =  
 (pushST [PrimT Boolean])  $\square$  (compTpExpr jmb G e)  $\square$  popST 2  $\square$   
 (compTpStmnt jmb G c1)  $\square$  nochangeST  $\square$  (compTpStmnt jmb G c2)"  
 | "compTpStmnt jmb G (While(e) c) =  
 (pushST [PrimT Boolean])  $\square$  (compTpExpr jmb G e)  $\square$  popST 2  $\square$   
 (compTpStmnt jmb G c)  $\square$  nochangeST"

```

definition compTpInit :: "java_mb  $\Rightarrow$  (vname * ty)
 $\Rightarrow$  state_type  $\Rightarrow$  method_type  $\times$  state_type" where
  "compTpInit jmb == ( $\lambda$  (vn,ty). (pushST [ty])  $\square$  (storeST (index jmb vn) ty))"

primrec compTpInitLvars :: "[java_mb, (vname  $\times$  ty) list]  $\Rightarrow$ 
  state_type  $\Rightarrow$  method_type  $\times$  state_type"
where
  "compTpInitLvars jmb [] = comb_nil"
| "compTpInitLvars jmb (lv#lvars) = (compTpInit jmb lv)  $\square$  (compTpInitLvars jmb lvars)"

definition start_ST :: "opstack_type" where
  "start_ST == []"

definition start_LT :: "cname  $\Rightarrow$  ty list  $\Rightarrow$  nat  $\Rightarrow$  locvars_type" where
  "start_LT C pTs n == (OK (Class C))#((map OK pTs))@(replicate n Err)"

definition compTpMethod :: "[java_mb prog, cname, java_mb mdecl]  $\Rightarrow$  method_type" where
  "compTpMethod G C ==  $\lambda$  ((mn,pTs),rT, jmb).
    let (pns,lvars,blk,res) = jmb
    in (mt_of
      ((compTpInitLvars jmb lvars  $\square$ 
        compTpStmt jmb G blk  $\square$ 
        compTpExpr jmb G res  $\square$ 
        nochangeST)
      (start_ST, start_LT C pTs (length lvars))))"

definition compTp :: "java_mb prog  $\Rightarrow$  prog_type" where
  "compTp G C sig == let (D, rT, jmb) = (the (method (G, C) sig))
    in compTpMethod G C (sig, rT, jmb)"

definition ssize_sto :: "(state_type option)  $\Rightarrow$  nat" where
  "ssize_sto sto == case sto of None  $\Rightarrow$  0 | (Some (ST, LT))  $\Rightarrow$  length ST"

definition max_of_list :: "nat list  $\Rightarrow$  nat" where
  "max_of_list xs == foldr max xs 0"

definition max_ssize :: "method_type  $\Rightarrow$  nat" where
  "max_ssize mt == max_of_list (map ssize_sto mt)"

end

theory TranslComp imports TranslCompTp begin

```



```

primrec compExpr :: "java_mb => expr => instr list"
  and compExprs :: "java_mb => expr list => instr list"
where

  "compExpr jmb (NewC c) = [New c]" |

  "compExpr jmb (Cast c e) = compExpr jmb e @ [Checkcast c]" |

  "compExpr jmb (Lit val) = [LitPush val]" |

  "compExpr jmb (BinOp bo e1 e2) = compExpr jmb e1 @ compExpr jmb e2 @
    (case bo of Eq => [Ifcmpeq 3, LitPush(Bool False), Goto 2, LitPush(Bool True)]
      | Add => [IAdd])" |

  "compExpr jmb (LAcc vn) = [Load (index jmb vn)]" |

  "compExpr jmb (vn ::= e) = compExpr jmb e @ [Dup , Store (index jmb vn)]" |

  "compExpr jmb ( {cn}e..fn ) = compExpr jmb e @ [Getfield fn cn]" |

  "compExpr jmb (FAss cn e1 fn e2 ) =
    compExpr jmb e1 @ compExpr jmb e2 @ [Dup_x1 , Putfield fn cn]" |

  "compExpr jmb (Call cn e1 mn X ps) =
    compExpr jmb e1 @ compExprs jmb ps @ [Invoke cn mn X]" |

  "compExprs jmb []      = []" |

  "compExprs jmb (e#es) = compExpr jmb e @ compExprs jmb es"

```

```
primrec compStmt :: "java_mb => stmt => instr list" where
```

```
"compStmt jmb Skip = []" |
"compStmt jmb (Expr e) = ((compExpr jmb e) @ [Pop])" |
"compStmt jmb (c1;; c2) = ((compStmt jmb c1) @ (compStmt jmb c2))" |
"compStmt jmb (If(e) c1 Else c2) =
  (let cnstf = LitPush (Bool False);
    cnd   = compExpr jmb e;
    thn   = compStmt jmb c1;
    els   = compStmt jmb c2;
    test  = Ifcmpeq (int(length thn +2));
    thnex = Goto (int(length els +1))
  in
  [cnstf] @ cnd @ [test] @ thn @ [thnex] @ els)" |
"compStmt jmb (While(e) c) =
  (let cnstf = LitPush (Bool False);
    cnd   = compExpr jmb e;
    bdy   = compStmt jmb c;
    test  = Ifcmpeq (int(length bdy +2));
    loop  = Goto (-(int((length bdy) + (length cnd) +2)))
  in
  [cnstf] @ cnd @ [test] @ bdy @ [loop])"
```

```
definition load_default_val :: "ty => instr" where
"load_default_val ty == LitPush (default_val ty)"
```

```
definition compInit :: "java_mb => (vname * ty) => instr list" where
"compInit jmb ==  $\lambda$  (vn,ty). [load_default_val ty, Store (index jmb vn)]"
```

```
definition compInitLvars :: "[java_mb, (vname  $\times$  ty) list]  $\Rightarrow$  bytecode" where
"compInitLvars jmb lvars == concat (map (compInit jmb) lvars)"
```

```
definition compMethod :: "java_mb prog  $\Rightarrow$  cname  $\Rightarrow$  java_mb mdecl  $\Rightarrow$  jvm_method mdecl" where
"compMethod G C jmdl == let (sig, rT, jmb) = jmdl;
  (pns,lvars,blk,res) = jmb;
  mt = (compTpMethod G C jmdl);
  bc = compInitLvars jmb lvars @
    compStmt jmb blk @ compExpr jmb res @
    [Return]
  in (sig, rT, max_ssize mt, length lvars, bc, [])"
```

```
definition compClass :: "java_mb prog => java_mb cdecl=> jvm_method cdecl" where
"compClass G ==  $\lambda$  (C,cno,fdls,jmdls). (C,cno,fdls, map (compMethod G C) jmdls)"
```

```

definition comp :: "java_mb prog => jvm_prog" where
  "comp G == map (compClass G) G"

```

```

end

```

```

theory LemmasComp
imports TranslComp
begin

```

```

context
begin

```

```

declare split_paired_All [simp del]
declare split_paired_Ex [simp del]

```

```

lemma c_hupd_conv:
  "c_hupd h' (xo, (h,l)) = (xo, (if xo = None then h' else h),l)"
  <proof>

```

```

lemma gl_c_hupd [simp]: "(gl (c_hupd h xs)) = (gl xs)"
  <proof>

```

```

lemma c_hupd_xcpt_invariant [simp]: "gx (c_hupd h' (xo, st)) = xo"
  <proof>

```

```

lemma c_hupd_hp_invariant: "gh (c_hupd hp (None, st)) = hp"
  <proof>

```

```

lemma unique_map_fst [rule_format]: "(∀ x ∈ set xs. (fst x = fst (f x) )) ⟶
  unique (map f xs) = unique xs"
  <proof>

```

```

lemma comp_unique: "unique (comp G) = unique G"
  <proof>

```

```

lemma comp_class_imp:
  "(class G C = Some(D, fs, ms)) ⟹
  (class (comp G) C = Some(D, fs, map (compMethod G C) ms))"
  <proof>

```

**lemma** *comp\_class\_None*:

"(class G C = None) = (class (comp G) C = None)"  
 ⟨proof⟩

**lemma** *comp\_is\_class*: "is\_class (comp G) C = is\_class G C"

⟨proof⟩

**lemma** *comp\_is\_type*: "is\_type (comp G) T = is\_type G T"

⟨proof⟩

**lemma** *comp\_classname*:

"is\_class G C  $\implies$  fst (the (class G C)) = fst (the (class (comp G) C))"  
 ⟨proof⟩

**lemma** *comp\_subcls1*: "subcls1 (comp G) = subcls1 G"

⟨proof⟩

**lemma** *comp\_widen*: "widen (comp G) = widen G"

⟨proof⟩

**lemma** *comp\_cast*: "cast (comp G) = cast G"

⟨proof⟩

**lemma** *comp\_cast\_ok*: "cast\_ok (comp G) = cast\_ok G"

⟨proof⟩

**lemma** *compClass\_fst [simp]*: "(fst (compClass G C)) = (fst C)"

⟨proof⟩

**lemma** *compClass\_fst\_snd [simp]*: "(fst (snd (compClass G C))) = (fst (snd C))"

⟨proof⟩

**lemma** *compClass\_fst\_snd\_snd [simp]*: "(fst (snd (snd (compClass G C)))) = (fst (snd (snd C)))"

⟨proof⟩

**lemma** *comp\_wf\_fdecl [simp]*: "wf\_fdecl (comp G) fd = wf\_fdecl G fd"

⟨proof⟩

**lemma** *compClass\_forall [simp]*:

"( $\forall x \in \text{set (snd (snd (snd (compClass G C))))}$ ). P (fst x) (fst (snd x))) =  
 ( $\forall x \in \text{set (snd (snd (snd C)))}$ ). P (fst x) (fst (snd x)))"  
 ⟨proof⟩

**lemma** *comp\_wf\_mhead*: "wf\_mhead (comp G) S rT = wf\_mhead G S rT"

⟨proof⟩

**lemma** *comp\_ws\_cdecl*:

"ws\_cdecl (TranslComp.comp G) (compClass G C) = ws\_cdecl G C"  
 ⟨proof⟩

```
lemma comp_wf_syscls: "wf_syscls (comp G) = wf_syscls G"
  <proof>
```

```
lemma comp_ws_prog: "ws_prog (comp G) = ws_prog G"
  <proof>
```

```
lemma comp_class_rec:
  "wf ((subcls1 G)-1) ⇒
  class_rec (comp G) C t f =
  class_rec G C t (λ C' fs' ms' r'. f C' fs' (map (compMethod G C') ms') r'))"
  <proof>
```

```
lemma comp_fields: "wf ((subcls1 G)-1) ⇒
  fields (comp G, C) = fields (G, C)"
  <proof>
```

```
lemma comp_field: "wf ((subcls1 G)-1) ⇒
  field (comp G, C) = field (G, C)"
  <proof>
```

```
lemma class_rec_relation [rule_format (no_asm)]: "[ ws_prog G;
  ∀ fs ms. R (f1 Object fs ms t1) (f2 Object fs ms t2);
  ∀ C fs ms r1 r2. (R r1 r2) → (R (f1 C fs ms r1) (f2 C fs ms r2)) ]
  ⇒ ((class G C) ≠ None) → R (class_rec G C t1 f1) (class_rec G C t2 f2)"
  <proof>
```

```
abbreviation (input)
  "mtd_mb == snd o snd"
```

```
lemma map_of_map:
  "map_of (map (λ(k, v). (k, f v)) xs) k = map_option f (map_of xs k)"
  <proof>
```

```
lemma map_of_map_fst:
  "[ inj f; ∀ x ∈ set xs. fst (f x) = fst x; ∀ x ∈ set xs. fst (g x) = fst x ]
  ⇒ map_of (map g xs) k = map_option (λ e. (snd (g ((inv f) (k, e))))) (map_of (map
  f xs) k)"
  <proof>
```

```
lemma comp_method [rule_format (no_asm)]:
  "[ ws_prog G; is_class G C ] ⇒
  ((method (comp G, C) S) =
  map_option (λ (D, rT, b). (D, rT, mtd_mb (compMethod G D (S, rT, b))))
  (method (G, C) S))"
  <proof>
```

```

lemma comp_wf_mrT: "[ ws_prog G; is_class G D ] ==>
  wf_mrT (TranslComp.comp G) (C, D, fs, map (compMethod G a) ms) =
  wf_mrT G (C, D, fs, ms)"
<proof>

```

```

lemma max_spec_preserves_length:
  "max_spec G C (mn, pTs) = {((md,rT),pTs')} ==> length pTs = length pTs'"
<proof>

```

```

lemma ty_exprs_length [simp]: "(E ⊢ es[::]Ts -> length es = length Ts)"
<proof>

```

```

lemma max_spec_preserves_method_rT [simp]:
  "max_spec G C (mn, pTs) = {((md,rT),pTs')}
  ==> method_rT (the (method (G, C) (mn, pTs')))) = rT"
<proof>

```

end

```

declare compClass_fst [simp del]
declare compClass_fst_snd [simp del]
declare compClass_fst_snd_snd [simp del]

```

end

```

theory CorrComp
imports "../J/JTypeSafe" LemmasComp
begin

```

```

declare wf_prog_ws_prog [simp add]

```

```

lemma eval_evals_exec_xcpt:
  "(G ⊢ xs -ex>val-> xs' -> gx xs' = None -> gx xs = None) ∧
  (G ⊢ xs -exs[>]vals-> xs' -> gx xs' = None -> gx xs = None) ∧
  (G ⊢ xs -st-> xs' -> gx xs' = None -> gx xs = None)"
<proof>

```

```

lemma eval_xcpt: "G ⊢ xs -ex>val-> xs' ==> gx xs' = None ==> gx xs = None"
(is "?H1 ==> ?H2 ==> ?T")

```

$\langle proof \rangle$

**lemma** evals\_xcpt: " $G \vdash xs \text{ -exs[>]vals-} \rightarrow xs' \implies gx\ xs' = None \implies gx\ xs = None$ "  
 (is "?H1  $\implies$  ?H2  $\implies$  ?T")

$\langle proof \rangle$

**lemma** exec\_xcpt: " $G \vdash xs \text{ -st-} \rightarrow xs' \implies gx\ xs' = None \implies gx\ xs = None$ "  
 (is "?H1  $\implies$  ?H2  $\implies$  ?T")

$\langle proof \rangle$

**theorem** exec\_all\_trans: " $\llbracket (\text{exec\_all } G\ s0\ s1); (\text{exec\_all } G\ s1\ s2) \rrbracket \implies (\text{exec\_all } G\ s0\ s2)$ "  
 $\langle proof \rangle$

**theorem** exec\_all\_refl: " $\text{exec\_all } G\ s\ s$ "  
 $\langle proof \rangle$

**theorem** exec\_instr\_in\_exec\_all:  
 " $\llbracket \text{exec\_instr } i\ G\ hp\ stk\ lvars\ C\ S\ pc\ frs = (None, hp', frs'); \text{gis } (gmb\ G\ C\ S) ! pc = i \rrbracket \implies$   
 $G \vdash (None, hp, (stk, lvars, C, S, pc) \# frs) \text{ -jvm-} \rightarrow (None, hp', frs')$ "  
 $\langle proof \rangle$

**theorem** exec\_all\_one\_step: "  
 $\llbracket \text{gis } (gmb\ G\ C\ S) = pre @ (i \# post); pc0 = \text{length } pre; (\text{exec\_instr } i\ G\ hp0\ stk0\ lvars0\ C\ S\ pc0\ frs) = (None, hp1, (stk1, lvars1, C, S, Suc\ pc0) \# frs) \rrbracket$   
 $\implies$   
 $G \vdash (None, hp0, (stk0, lvars0, C, S, pc0) \# frs) \text{ -jvm-} \rightarrow (None, hp1, (stk1, lvars1, C, S, Suc\ pc0) \# frs)$ "  
 $\langle proof \rangle$

**definition** progression :: " $\text{jvm\_prog} \Rightarrow \text{cname} \Rightarrow \text{sig} \Rightarrow$   
 $\text{aheap} \Rightarrow \text{opstack} \Rightarrow \text{locvars} \Rightarrow$   
 $\text{bytecode} \Rightarrow$   
 $\text{aheap} \Rightarrow \text{opstack} \Rightarrow \text{locvars} \Rightarrow$   
 $\text{bool}$ "  
 ( $\langle \{_, \_, \_ \} \vdash \{_, \_, \_ \} \text{ >- } \_ \rightarrow \{_, \_, \_ \} \text{ [61,61,61,61,61,61,90,61,61,61]60} \rangle$  where  
 " $\{G, C, S\} \vdash \{hp0, os0, lvars0\} \text{ >- instrs} \rightarrow \{hp1, os1, lvars1\} ==$   
 $\forall pre\ post\ frs.$   
 $(\text{gis } (gmb\ G\ C\ S) = pre @ instrs @ post) \longrightarrow$   
 $G \vdash (None, hp0, (os0, lvars0, C, S, \text{length } pre) \# frs) \text{ -jvm-} \rightarrow$   
 $(None, hp1, (os1, lvars1, C, S, (\text{length } pre) + (\text{length } instrs)) \# frs)$ "

**lemma** progression\_call:

```

"[[  $\forall pc \text{ frs.}$ 
exec_instr instr G hp0 os0 lvars0 C S pc frs =
  (None, hp', (os', lvars', C', S', 0) # (fr pc) # frs)  $\wedge$ 
gis (gmb G C' S') = instrs' @ [Return]  $\wedge$ 
{G, C', S'}  $\vdash$  {hp', os', lvars'}  $\text{>- instrs'}$   $\rightarrow$  {hp'', os'', lvars''}  $\wedge$ 
exec_instr Return G hp'' os'' lvars'' C' S' (length instrs')
  ((fr pc) # frs) =
  (None, hp2, (os2, lvars2, C, S, Suc pc) # frs) ]]  $\Rightarrow$ 
{G, C, S}  $\vdash$  {hp0, os0, lvars0}  $\text{>- [instr]}$   $\rightarrow$  {hp2, os2, lvars2}"
<proof>

```

**lemma progression\_transitive:**

```

"[[ instrs_comb = instrs0 @ instrs1;
{G, C, S}  $\vdash$  {hp0, os0, lvars0}  $\text{>- instrs0}$   $\rightarrow$  {hp1, os1, lvars1};
{G, C, S}  $\vdash$  {hp1, os1, lvars1}  $\text{>- instrs1}$   $\rightarrow$  {hp2, os2, lvars2} ]]
 $\Rightarrow$ 
{G, C, S}  $\vdash$  {hp0, os0, lvars0}  $\text{>- instrs_comb}$   $\rightarrow$  {hp2, os2, lvars2}"
<proof>

```

**lemma progression\_refl:**

```

"{G, C, S}  $\vdash$  {hp0, os0, lvars0}  $\text{>- []}$   $\rightarrow$  {hp0, os0, lvars0}"
<proof>

```

**lemma progression\_one\_step:** "

```

 $\forall pc \text{ frs.}$ 
(exec_instr i G hp0 os0 lvars0 C S pc frs) =
  (None, hp1, (os1, lvars1, C, S, Suc pc) # frs)
 $\Rightarrow$  {G, C, S}  $\vdash$  {hp0, os0, lvars0}  $\text{>- [i]}$   $\rightarrow$  {hp1, os1, lvars1}"
<proof>

```

**definition jump\_fwd :: "jvm\_prog  $\Rightarrow$  cname  $\Rightarrow$  sig  $\Rightarrow$**

**aheap  $\Rightarrow$  locvars  $\Rightarrow$  opstack  $\Rightarrow$  opstack  $\Rightarrow$**   
**instr  $\Rightarrow$  bytecode  $\Rightarrow$  bool" where**

```

"jump_fwd G C S hp lvars os0 os1 instr instrs ==
 $\forall pre \text{ post frs.}$ 
(gis (gmb G C S) = pre @ instr # instrs @ post)  $\rightarrow$ 
exec_all G (None, hp, (os0, lvars, C, S, length pre) # frs)
  (None, hp, (os1, lvars, C, S, (length pre) + (length instrs) + 1) # frs)"

```

**lemma jump\_fwd\_one\_step:**

```

" $\forall pc \text{ frs.}$ 
exec_instr instr G hp os0 lvars C S pc frs =
  (None, hp, (os1, lvars, C, S, pc + (length instrs) + 1) # frs)
 $\Rightarrow$  jump_fwd G C S hp lvars os0 os1 instr instrs"
<proof>

```

**lemma jump\_fwd\_progression\_aux:**

```

"[[ instrs_comb = instr # instrs0 @ instrs1;
jump_fwd G C S hp lvars os0 os1 instr instrs0;
{G, C, S}  $\vdash$  {hp, os1, lvars}  $\text{>- instrs1}$   $\rightarrow$  {hp2, os2, lvars2} ]]
```



$\Rightarrow \{G, C, S\} \vdash \{hp, os0, lvars\} \text{>- instrs\_comb} \rightarrow \{hp2, os2, lvars2\}$ "  
 $\langle proof \rangle$

**lemma** *jump\_fwd\_progression*:

"[ instrs\_comb = instr # instrs0 @ instrs1;  
 $\forall pc\ frs.$   
 $exec\_instr\ instr\ G\ hp\ os0\ lvars\ C\ S\ pc\ frs =$   
 $(None, hp, (os1, lvars, C, S, pc + (length\ instrs0) + 1)\#frs);$   
 $\{G, C, S\} \vdash \{hp, os1, lvars\} \text{>- instrs1} \rightarrow \{hp2, os2, lvars2\}$  ]  
 $\Rightarrow \{G, C, S\} \vdash \{hp, os0, lvars\} \text{>- instrs\_comb} \rightarrow \{hp2, os2, lvars2\}$ "  
 $\langle proof \rangle$

**definition** *jump\_bwd* :: "jvm\_prog  $\Rightarrow$  cname  $\Rightarrow$  sig  $\Rightarrow$

aheap  $\Rightarrow$  locvars  $\Rightarrow$  opstack  $\Rightarrow$  opstack  $\Rightarrow$   
 bytecode  $\Rightarrow$  instr  $\Rightarrow$  bool" **where**

"jump\_bwd G C S hp lvars os0 os1 instrs instr ==  
 $\forall pre\ post\ frs.$   
 $(gis\ (gmb\ G\ C\ S) = pre\ @\ instrs\ @\ instr\ \# post) \rightarrow$   
 $exec\_all\ G\ (None, hp, (os0, lvars, C, S, (length\ pre) + (length\ instrs))\#frs)$   
 $(None, hp, (os1, lvars, C, S, (length\ pre))\#frs)"$

**lemma** *jump\_bwd\_one\_step*:

" $\forall pc\ frs.$   
 $exec\_instr\ instr\ G\ hp\ os0\ lvars\ C\ S\ (pc + (length\ instrs))\ frs =$   
 $(None, hp, (os1, lvars, C, S, pc)\#frs)$   
 $\Rightarrow$   
 $jump\_bwd\ G\ C\ S\ hp\ lvars\ os0\ os1\ instrs\ instr"$   
 $\langle proof \rangle$

**lemma** *jump\_bwd\_progression*:

"[ instrs\_comb = instrs @ [instr];  
 $\{G, C, S\} \vdash \{hp0, os0, lvars0\} \text{>- instrs} \rightarrow \{hp1, os1, lvars1\};$   
 $jump\_bwd\ G\ C\ S\ hp1\ lvars1\ os1\ os2\ instrs\ instr;$   
 $\{G, C, S\} \vdash \{hp1, os2, lvars1\} \text{>- instrs\_comb} \rightarrow \{hp3, os3, lvars3\}$  ]  
 $\Rightarrow \{G, C, S\} \vdash \{hp0, os0, lvars0\} \text{>- instrs\_comb} \rightarrow \{hp3, os3, lvars3\}$ "  
 $\langle proof \rangle$

**definition** *class\_sig\_defined* :: "'c prog  $\Rightarrow$  cname  $\Rightarrow$  sig  $\Rightarrow$  bool" **where**

"class\_sig\_defined G C S ==  
 $is\_class\ G\ C \wedge (\exists D\ rT\ mb. (method\ (G, C)\ S = Some\ (D, rT, mb)))"$

**definition** *env\_of\_jmb* :: "java\_mb prog  $\Rightarrow$  cname  $\Rightarrow$  sig  $\Rightarrow$  java\_mb env" **where**

"env\_of\_jmb G C S ==  
 $(let\ (mn, pTs) = S;$

```
(D,rT,(pns,lvars,blk,res)) = the(method (G, C) S) in
(G,(map_of lvars)(pns[↦]pTs, This↦Class C)))"
```

```
lemma env_of_jmbfst [simp]: "fst (env_of_jmb G C S) = G"
⟨proof⟩
```

```
lemma method_preserves [rule_format (no_asm)]:
  "[ wf_prog wf_mb G; is_class G C;
    ∀ S rT mb. ∀ cn ∈ fst ' set G. wf_mdecl wf_mb G cn (S,rT,mb)  ⟶ (P cn S (rT,mb)) ]
  ⟹ ∀ D.
    method (G, C) S = Some (D, rT, mb) ⟶ (P D S (rT,mb))"

⟨proof⟩
```

```
lemma method_preserves_length:
  "[ wf_java_prog G; is_class G C;
    method (G, C) (mn,pTs) = Some (D, rT, pns, lvars, blk, res) ]
  ⟹ length pns = length pTs"
⟨proof⟩
```

```
definition wtpd_expr :: "java_mb env ⇒ expr ⇒ bool" where
  "wtpd_expr E e == (∃ T. E⊢e :: T)"
```

```
definition wtpd_exprs :: "java_mb env ⇒ (expr list) ⇒ bool" where
  "wtpd_exprs E e == (∃ T. E⊢e [::] T)"
```

```
definition wtpd_stmt :: "java_mb env ⇒ stmt ⇒ bool" where
  "wtpd_stmt E c == (E⊢c √)"
```

```
lemma wtpd_expr_newc: "wtpd_expr E (NewC C) ⟹ is_class (prg E) C"
⟨proof⟩
```

```
lemma wtpd_expr_cast: "wtpd_expr E (Cast cn e) ⟹ (wtpd_expr E e)"
⟨proof⟩
```

```
lemma wtpd_expr_lacc:
  "[ wtpd_expr (env_of_jmb G C S) (LAcc vn); class_sig_defined G C S ]
  ⟹ vn ∈ set (gjmb_plns (gmb G C S)) ∨ vn = This"
⟨proof⟩
```

```
lemma wtpd_expr_lass: "wtpd_expr E (vn::=e)
  ⟹ (vn ≠ This) & (wtpd_expr E (LAcc vn)) & (wtpd_expr E e)"
⟨proof⟩
```

```
lemma wtpd_expr_facc: "wtpd_expr E ({fd}a..fn)
  ⟹ (wtpd_expr E a)"
⟨proof⟩
```

```
lemma wtpd_expr_fass: "wtpd_expr E ({fd}a..fn:=v)
  ⇒ (wtpd_expr E ({fd}a..fn)) & (wtpd_expr E v)"
  ⟨proof⟩
```

```
lemma wtpd_expr_binop: "wtpd_expr E (BinOp bop e1 e2)
  ⇒ (wtpd_expr E e1) & (wtpd_expr E e2)"
  ⟨proof⟩
```

```
lemma wtpd_exprs_cons: "wtpd_exprs E (e # es)
  ⇒ (wtpd_expr E e) & (wtpd_exprs E es)"
  ⟨proof⟩
```

```
lemma wtpd_stmt_expr: "wtpd_stmt E (Expr e) ⇒ (wtpd_expr E e)"
  ⟨proof⟩
```

```
lemma wtpd_stmt_comp: "wtpd_stmt E (s1;; s2) ⇒
  (wtpd_stmt E s1) & (wtpd_stmt E s2)"
  ⟨proof⟩
```

```
lemma wtpd_stmt_cond: "wtpd_stmt E (If(e) s1 Else s2) ⇒
  (wtpd_expr E e) & (wtpd_stmt E s1) & (wtpd_stmt E s2)
  & (E⊢e::PrimT Boolean)"
  ⟨proof⟩
```

```
lemma wtpd_stmt_loop: "wtpd_stmt E (While(e) s) ⇒
  (wtpd_expr E e) & (wtpd_stmt E s) & (E⊢e::PrimT Boolean)"
  ⟨proof⟩
```

```
lemma wtpd_expr_call: "wtpd_expr E ({C}a..mn({pTs'}ps))
  ⇒ (wtpd_expr E a) & (wtpd_exprs E ps)
  & (length ps = length pTs') & (E⊢a::Class C)
  & (∃ pTs md rT.
    E⊢ps[::]pTs & max_spec (prg E) C (mn, pTs) = {((md,rT),pTs')})"
  ⟨proof⟩
```

```
lemma wtpd_blk:
  "[[ method (G, D) (md, pTs) = Some (D, rT, (pns, lvars, blk, res));
    wf_prog wf_java_mdecl G; is_class G D ]]"
  ⇒ wtpd_stmt (env_of_jmb G D (md, pTs)) blk"
  ⟨proof⟩
```

```
lemma wtpd_res:
  "[[ method (G, D) (md, pTs) = Some (D, rT, (pns, lvars, blk, res));
    wf_prog wf_java_mdecl G; is_class G D ]]"
  ⇒ wtpd_expr (env_of_jmb G D (md, pTs)) res"
  ⟨proof⟩
```

**lemma** *evals\_preserves\_length*:

" $G \vdash xs \text{ -es[>] } vs \rightarrow (None, s) \implies \text{length } es = \text{length } vs$ "  
 <proof>

**lemma** *progression\_Eq* : " $\{G, C, S\} \vdash$

{hp, (v2 # v1 # os), lvars}  
 >- [Ifcmpeq 3, LitPush (Bool False), Goto 2, LitPush (Bool True)]  $\rightarrow$   
 {hp, (Bool (v1 = v2) # os), lvars}"  
 <proof>

**declare** *split\_paired\_All* [simp del] *split\_paired\_Ex* [simp del]

**lemma** *distinct\_method*:

"[ wf\_java\_prog G; is\_class G C; method (G, C) S = Some (D, rT, pns, lvars, blk, res) ]  
 ]  $\implies$   
 distinct (gjmb\_plns (gmb G C S))"  
 <proof>

**lemma** *distinct\_method\_if\_class\_sig\_defined* :

"[ wf\_java\_prog G; class\_sig\_defined G C S ]  $\implies$  distinct (gjmb\_plns (gmb G C S))"  
 <proof>

**lemma** *method\_yields\_wf\_java\_mdecl*: "[ wf\_java\_prog G; is\_class G C;

method (G, C) S = Some (D, rT, pns, lvars, blk, res) ]  $\implies$   
 wf\_java\_mdecl G D (S, rT, (pns, lvars, blk, res))"  
 <proof>

**lemma** *progression\_lvar\_init\_aux* [rule\_format (no\_asm)]: "

$\forall zs \text{ prfx } lvals \text{ lvars0.}$   
 $lvars0 = (zs @ lvars) \rightarrow$   
 $(\text{disjoint\_varnames } pns \text{ lvars0} \rightarrow$   
 $(\text{length } lvars = \text{length } lvals) \rightarrow$   
 $(\text{Suc}(\text{length } pns + \text{length } zs) = \text{length } prfx) \rightarrow$   
 $\{cG, D, S\} \vdash$   
 $\{h, os, (prfx @ lvals)\}$   
 $>- (\text{concat } (\text{map } (\text{compInit } (pns, lvars0, blk, res)) \text{ lvars})) \rightarrow$   
 $\{h, os, (prfx @ (\text{map } (\lambda p. (\text{default\_val } (snd p))) \text{ lvars}))\})$ "  
 <proof>

```

lemma progression_lvar_init [rule_format (no_asm)]:
  "[ wf_java_prog G; is_class G C;
    method (G, C) S = Some (D, rT, pns, lvars, blk, res) ] ==>
    length pns = length pvs ->
    (∀ lvals.
    length lvars = length lvals ->
    {cG, D, S} ⊢
    {h, os, (a' # pvs @ lvals)}
    >- (compInitLvars (pns, lvars, blk, res) lvars) ->
    {h, os, (locvars_xstate G C S (Norm (h, (init_vars lvars)(pns[↦]pvs, This↦a'))))})"
  <proof>

```

```

lemma state_ok_eval:
  "[xs::≤E; wf_java_prog (prg E); wtpd_expr E e; (prg E) ⊢ xs -e>v -> xs'] ==> xs'::≤E"
  <proof>

```

```

lemma state_ok_evals:
  "[xs::≤E; wf_java_prog (prg E); wtpd_exprs E es; prg E ⊢ xs -es[>]vs-> xs'] ==> xs'::≤E"
  <proof>

```

```

lemma state_ok_exec:
  "[xs::≤E; wf_java_prog (prg E); wtpd_stmt E st; prg E ⊢ xs -st-> xs'] ==> xs'::≤E"
  <proof>

```

```

lemma state_ok_init:
  "[ wf_java_prog G; (x, h, l)::≤(env_of_jmb G C S);
    is_class G dynT;
    method (G, dynT) (mn, pTs) = Some (md, rT, pns, lvars, blk, res);
    list_all2 (conf G h) pvs pTs; G, h ⊢ a' ::≤ Class md ]
  ==>
  (np a' x, h, (init_vars lvars)(pns[↦]pvs, This↦a'))::≤(env_of_jmb G md (mn, pTs))"
  <proof>

```

```

lemma ty_exprs_list_all2 [rule_format (no_asm)]:
  "(∀ Ts. (E ⊢ es [::] Ts) = list_all2 (λe T. E ⊢ e :: T) es Ts)"
  <proof>

```

```

lemma conf_bool: "G, h ⊢ v::≤PrimT Boolean ==> ∃ b. v = Bool b"
  <proof>

```

```

lemma max_spec_widen: "max_spec G C (mn, pTs) = {(md, rT), pTs'} ==>
  list_all2 (λ T T'. G ⊢ T ≤ T') pTs pTs'"
  <proof>

```

```

lemma eval_conf: "[G ⊢ s -e>v-> s'; wf_java_prog G; s::≤E;

```

$E \vdash e :: T; \text{gx } s' = \text{None}; \text{prg } E = G \parallel$   
 $\implies G, \text{gh } s' \vdash v :: \preceq T$   
 $\langle \text{proof} \rangle$

**lemma evals\_preserves\_conf:**  
 $"\parallel G \vdash s \text{ -es}[\succ] \text{vs} \rightarrow s'; G, \text{gh } s \vdash t :: \preceq T; E \vdash \text{es}[:, :] Ts;$   
 $\text{wf\_java\_prog } G; s :: \preceq E;$   
 $\text{prg } E = G \parallel \implies G, \text{gh } s' \vdash t :: \preceq T"$   
 $\langle \text{proof} \rangle$

**lemma eval\_of\_class:**  
 $"\parallel G \vdash s \text{ -e} \succ a' \rightarrow s'; E \vdash e :: \text{Class } C; \text{wf\_java\_prog } G; s :: \preceq E; \text{gx } s' = \text{None}; a' \neq \text{Null};$   
 $G = \text{prg } E \parallel$   
 $\implies (\exists \text{ lc. } a' = \text{Addr lc})"$   
 $\langle \text{proof} \rangle$

**lemma dynT\_subcls:**  
 $"\parallel a' \neq \text{Null}; G, h \vdash a' :: \preceq \text{Class } C; \text{dynT} = \text{fst } (\text{the } (h \text{ (the\_Addr } a'))));$   
 $\text{is\_class } G \text{ dynT}; \text{ws\_prog } G \parallel \implies G \vdash \text{dynT} \preceq_C C"$   
 $\langle \text{proof} \rangle$

**lemma method\_defined:** " $\parallel$   
 $m = \text{the } (\text{method } (G, \text{dynT}) (mn, pTs));$   
 $\text{dynT} = \text{fst } (\text{the } (h \text{ } a)); \text{is\_class } G \text{ dynT}; \text{wf\_java\_prog } G;$   
 $a' \neq \text{Null}; G, h \vdash a' :: \preceq \text{Class } C; a = \text{the\_Addr } a';$   
 $\exists pTsa \text{ md } rT. \text{max\_spec } G \text{ } C (mn, pTsa) = \{((md, rT), pTs)\} \parallel$   
 $\implies (\text{method } (G, \text{dynT}) (mn, pTs)) = \text{Some } m"$   
 $\langle \text{proof} \rangle$

**theorem compiler\_correctness:**  
 $"\text{wf\_java\_prog } G \implies$   
 $(G \vdash xs \text{ -ex} \succ \text{val} \rightarrow xs' \rightarrow$   
 $\text{gx } xs = \text{None} \rightarrow \text{gx } xs' = \text{None} \rightarrow$   
 $(\forall \text{ os CL S.}$   
 $(\text{class\_sig\_defined } G \text{ CL S}) \rightarrow$   
 $(\text{wtpd\_expr } (\text{env\_of\_jmb } G \text{ CL S}) \text{ ex}) \rightarrow$   
 $(xs :: \preceq (\text{env\_of\_jmb } G \text{ CL S})) \rightarrow$   
 $(\{ \text{TranslComp.comp } G, \text{CL}, S \} \vdash$   
 $\{ \text{gh } xs, \text{os}, (\text{locvars\_xstate } G \text{ CL S } xs) \}$   
 $\succ - (\text{compExpr } (\text{gmb } G \text{ CL S}) \text{ ex}) \rightarrow$   
 $\{ \text{gh } xs', \text{val\#os}, (\text{locvars\_xstate } G \text{ CL S } xs') \}))) \wedge$   
 $(G \vdash xs \text{ -exs}[\succ] \text{vals} \rightarrow xs' \rightarrow$   
 $\text{gx } xs = \text{None} \rightarrow \text{gx } xs' = \text{None} \rightarrow$   
 $(\forall \text{ os CL S.}$

```

(class_sig_defined G CL S) →
(wtpd_exprs (env_of_jmb G CL S) exs) →
(xs ::  $\preceq$  (env_of_jmb G CL S)) →
( {TranslComp.comp G, CL, S} ⊢
  {gh xs, os, (locvars_xstate G CL S xs)}
  >- (compExprs (gmb G CL S) exs) →
  {gh xs', (rev vals)@os, (locvars_xstate G CL S xs')}})) ∧

(G ⊢ xs -st-> xs' →
  gx xs = None → gx xs' = None →
  (∀ os CL S.
    (class_sig_defined G CL S) →
    (wtpd_stmt (env_of_jmb G CL S) st) →
    (xs ::  $\preceq$  (env_of_jmb G CL S)) →
    ( {TranslComp.comp G, CL, S} ⊢
      {gh xs, os, (locvars_xstate G CL S xs)}
      >- (compStmt (gmb G CL S) st) →
      {gh xs', os, (locvars_xstate G CL S xs')}}))"
  <proof>

```

```

theorem compiler_correctness_eval: "
  [ G ⊢ (None, hp, loc) -ex > val-> (None, hp', loc');
  wf_java_prog G;
  class_sig_defined G C S;
  wtpd_expr (env_of_jmb G C S) ex;
  (None, hp, loc) ::  $\preceq$  (env_of_jmb G C S) ] ⇒
  {(TranslComp.comp G), C, S} ⊢
  {hp, os, (locvars_locals G C S loc)}
  >- (compExpr (gmb G C S) ex) →
  {hp', val#os, (locvars_locals G C S loc')}"
  <proof>

```

```

theorem compiler_correctness_exec: "
  [ G ⊢ Norm (hp, loc) -st-> Norm (hp', loc');
  wf_java_prog G;
  class_sig_defined G C S;
  wtpd_stmt (env_of_jmb G C S) st;
  (None, hp, loc) ::  $\preceq$  (env_of_jmb G C S) ] ⇒
  {(TranslComp.comp G), C, S} ⊢
  {hp, os, (locvars_locals G C S loc)}
  >- (compStmt (gmb G C S) st) →
  {hp', os, (locvars_locals G C S loc')}"
  <proof>

```

```

declare split_paired_All [simp] split_paired_Ex [simp]

```

```

declare wf_prog_ws_prog [simp del]

```

```

end

```

```

theory TypeInf
imports "../J/WellType"
begin

```

```

lemma NewC_invers:
  assumes "E ⊢ NewC C :: T"
  shows "T = Class C ∧ is_class (prg E) C"
  ⟨proof⟩

```

```

lemma Cast_invers:
  assumes "E ⊢ Cast D e :: T"
  shows "∃ C. T = Class D ∧ E ⊢ e :: C ∧ is_class (prg E) D ∧ prg E ⊢ C ≤? Class D"
  ⟨proof⟩

```

```

lemma Lit_invers:
  assumes "E ⊢ Lit x :: T"
  shows "typeof (λv. None) x = Some T"
  ⟨proof⟩

```

```

lemma LAcc_invers:
  assumes "E ⊢ LAcc v :: T"
  shows "localT E v = Some T ∧ is_type (prg E) T"
  ⟨proof⟩

```

```

lemma BinOp_invers:
  assumes "E ⊢ BinOp bop e1 e2 :: T'"
  shows "∃ T. E ⊢ e1 :: T ∧ E ⊢ e2 :: T ∧
    (if bop = Eq then T' = PrimT Boolean
     else T' = T ∧ T = PrimT Integer)"
  ⟨proof⟩

```

```

lemma LAss_invers:
  assumes "E ⊢ v := e :: T'"
  shows "∃ T. v ~= This ∧ E ⊢ LAcc v :: T ∧ E ⊢ e :: T' ∧ prg E ⊢ T' ≤ T"
  ⟨proof⟩

```

```

lemma FAcc_invers:
  assumes "E ⊢ {fd} a .. fn :: fT"
  shows "∃ C. E ⊢ a :: Class C ∧ field (prg E, C) fn = Some (fd, fT)"
  ⟨proof⟩

```

```

lemma FAss_invers:
  assumes "E ⊢ {fd} a .. fn := v :: T'"
  shows "∃ T. E ⊢ {fd} a .. fn :: T ∧ E ⊢ v :: T' ∧ prg E ⊢ T' ≤ T"

```



$\langle proof \rangle$

**lemma** *Call\_invers*:

assumes " $E \vdash \{C\} a. .mn(\{pTs'\}ps) :: rT$ "

shows " $\exists pTs \text{ md.}$ "

$E \vdash a :: \text{Class } C \wedge E \vdash ps[::]pTs \wedge \text{max\_spec } (prg \ E) \ C \ (mn, \ pTs) = \{((md, rT), pTs')\}$ "

$\langle proof \rangle$

**lemma** *Nil\_invers*:

assumes " $E \vdash [] [::] Ts$ "

shows " $Ts = []$ "

$\langle proof \rangle$

**lemma** *Cons\_invers*:

assumes " $E \vdash e \# es[::]Ts$ "

shows " $\exists T \ Ts'. \ Ts = T \# Ts' \wedge E \vdash e :: T \wedge E \vdash es[::]Ts'$ "

$\langle proof \rangle$

**lemma** *Expr\_invers*:

assumes " $E \vdash \text{Expr } e \checkmark$ "

shows " $\exists T. \ E \vdash e :: T$ "

$\langle proof \rangle$

**lemma** *Comp\_invers*:

assumes " $E \vdash s1;; s2 \checkmark$ "

shows " $E \vdash s1 \checkmark \wedge E \vdash s2 \checkmark$ "

$\langle proof \rangle$

**lemma** *Cond\_invers*:

assumes " $E \vdash \text{If}(e) \ s1 \ \text{Else } s2 \checkmark$ "

shows " $E \vdash e :: \text{PrimT Boolean} \wedge E \vdash s1 \checkmark \wedge E \vdash s2 \checkmark$ "

$\langle proof \rangle$

**lemma** *Loop\_invers*:

assumes " $E \vdash \text{While}(e) \ s \checkmark$ "

shows " $E \vdash e :: \text{PrimT Boolean} \wedge E \vdash s \checkmark$ "

$\langle proof \rangle$

**declare** *split\_paired\_All* [*simp del*]

**declare** *split\_paired\_Ex* [*simp del*]

**method** *ty\_case\_simp* = ((*erule ty\_exprs.cases ty\_expr.cases; simp*)+) []

**method** *strip\_case\_simp* = (*intro strip, ty\_case\_simp*)

**lemma** *uniqueness\_of\_types*: "

$(\forall (E :: 'a \text{ prog} \times (\text{vname} \Rightarrow \text{ty option}))) \ T1 \ T2.$

```


$$E \vdash e :: T1 \longrightarrow E \vdash e :: T2 \longrightarrow T1 = T2) \wedge$$


$$(\forall (E :: 'a \text{ prog} \times (\text{vname} \Rightarrow \text{ty option})) Ts1 Ts2.$$


$$E \vdash \text{es} [::] Ts1 \longrightarrow E \vdash \text{es} [::] Ts2 \longrightarrow Ts1 = Ts2)"$$


$$\langle \text{proof} \rangle$$


lemma uniqueness_of_types_expr [rule_format (no_asm)]: "
  ( $\forall E T1 T2. E \vdash e :: T1 \longrightarrow E \vdash e :: T2 \longrightarrow T1 = T2$ )"
   $\langle \text{proof} \rangle$ 

lemma uniqueness_of_types_exprs [rule_format (no_asm)]: "
  ( $\forall E Ts1 Ts2. E \vdash \text{es} [::] Ts1 \longrightarrow E \vdash \text{es} [::] Ts2 \longrightarrow Ts1 = Ts2$ )"
   $\langle \text{proof} \rangle$ 

definition inferred_tp :: "[java_mb env, expr]  $\Rightarrow$  ty" where
  "inferred_tp E e == (SOME T.  $E \vdash e :: T$ )"

definition inferred_tps :: "[java_mb env, expr list]  $\Rightarrow$  ty list" where
  "inferred_tps E es == (SOME Ts.  $E \vdash \text{es} [::] Ts$ )"

lemma inferred_tp_wt: " $E \vdash e :: T \implies (\text{inferred\_tp } E \text{ } e) = T$ "
   $\langle \text{proof} \rangle$ 

lemma inferred_tps_wt: " $E \vdash \text{es} [::] Ts \implies (\text{inferred\_tps } E \text{ } es) = Ts$ "
   $\langle \text{proof} \rangle$ 

end

```

## 4.27 Alternative definition of well-typing of bytecode, used in compiler type correctness proof

```

theory Altern
imports BVSpec
begin

definition check_type :: "jvm_prog  $\Rightarrow$  nat  $\Rightarrow$  nat  $\Rightarrow$  JVMType.state  $\Rightarrow$  bool" where
  "check_type G mxs mxr s  $\equiv$   $s \in \text{states } G \text{ mxs mxr}$ "

definition wt_instr_altern :: "[instr, jvm_prog, ty, method_type, nat, nat, p_count,
  exception_table, p_count]  $\Rightarrow$  bool" where
  "wt_instr_altern i G rT phi mxs mxr max_pc et pc  $\equiv$ 
  app i G mxs rT pc et (phi!pc)  $\wedge$ 
  check_type G mxs mxr (OK (phi!pc))  $\wedge$ 
  ( $\forall (pc', s') \in \text{set } (\text{eff } i \text{ } G \text{ } pc \text{ } et \text{ } (phi!pc)). pc' < \text{max\_pc} \wedge G \vdash s' <= 'phi!pc')$ "

definition wt_method_altern :: "[jvm_prog, cname, ty list, ty, nat, nat, instr list,
  exception_table, method_type]  $\Rightarrow$  bool" where
  "wt_method_altern G C pTs rT mxs mxl ins et phi  $\equiv$ 
  let max_pc = length ins in
  0 < max_pc  $\wedge$ 
  length phi = length ins  $\wedge$ 

```

```

  check_bounded ins et ∧
  wt_start G C pTs mxl phi ∧
  (∀ pc. pc < max_pc → wt_instr_altern (ins!pc) G rT phi mxs (1+length pTs+mxl) max_pc
  et pc)"

```

```

lemma wt_method_wt_method_altern :
  "wt_method G C pTs rT mxs mxl ins et phi → wt_method_altern G C pTs rT mxs mxl ins
  et phi"
  ⟨proof⟩

```

```

lemma check_type_check_types [rule_format]:
  "(∀ pc. pc < length phi → check_type G mxs mxr (OK (phi ! pc)))
  → check_types G mxs mxr (map OK phi)"
  ⟨proof⟩

```

```

lemma wt_method_altern_wt_method [rule_format]:
  "wt_method_altern G C pTs rT mxs mxl ins et phi → wt_method G C pTs rT mxs mxl ins
  et phi"
  ⟨proof⟩

```

**end**

```

theory CorrCompTp
imports LemmasComp TypeInf "../BV/JVM" "../BV/Altern"
begin

```

```

declare split_paired_All [simp del]
declare split_paired_Ex [simp del]

```

```

definition inited_LT :: "[cname, ty list, (vname × ty) list] ⇒ locvars_type" where
  "inited_LT C pTs lvars == (OK (Class C))#((map OK pTs))@(map (Fun.comp OK snd) lvars)"

```

```

definition is_inited_LT :: "[cname, ty list, (vname × ty) list, locvars_type] ⇒ bool"
where
  "is_inited_LT C pTs lvars LT == (LT = (inited_LT C pTs lvars))"

```

```

definition local_env :: "[java_mb prog, cname, sig, vname list, (vname × ty) list] ⇒ java_mb
  env" where
  "local_env G C S pns lvars ==
  let (mn, pTs) = S in (G, (map_of lvars) (pns[↦] pTs, This ↦ Class C))"

```

```

lemma local_env_fst [simp]: "fst (local_env G C S pns lvars) = G"
  ⟨proof⟩

```

```

lemma wt_class_expr_is_class:
  "[ ws_prog G; E ⊢ expr :: Class cname; E = local_env G C (mn, pTs) pns lvars ]

```

```

     $\Rightarrow$  is_class G cname "
    <proof>

```

#### 4.27.1 index

```

lemma local_env_snd:
  "snd (local_env G C (mn, pTs) pns lvars) = (map_of lvars) (pns [↦] pTs, This ↦ Class C)"
  <proof>

```

```

lemma index_in_bounds:
  "length pns = length pTs  $\Rightarrow$ 
  snd (local_env G C (mn, pTs) pns lvars) vname = Some T
   $\Rightarrow$  index (pns, lvars, blk, res) vname < length (inited_LT C pTs lvars)"
  <proof>

```

```

lemma map_upds_append:
  "length k1s = length x1s  $\Rightarrow$  m(k1s [↦] x1s, k2s [↦] x2s) = m ((k1s @ k2s) [↦] (x1s @ x2s))"
  <proof>

```

```

lemma map_of_append:
  "map_of ((rev xs) @ ys) = (map_of ys) ((map fst xs) [↦] (map snd xs))"
  <proof>

```

```

lemma map_of_as_map_upds: "map_of (rev xs) = Map.empty ((map fst xs) [↦] (map snd xs))"
  <proof>

```

```

lemma map_of_rev: "unique xs  $\Rightarrow$  map_of (rev xs) = map_of xs"
  <proof>

```

```

lemma map_upds_rev:
  "[[ distinct ks; length ks = length xs ]  $\Rightarrow$  m (rev ks [↦] rev xs) = m (ks [↦] xs)]"
  <proof>

```

```

lemma map_upds_takeWhile [rule_format]:
  " $\forall$  ks. (Map.empty (rev ks [↦] rev xs)) k = Some x  $\longrightarrow$  length ks = length xs  $\longrightarrow$ 
  xs ! length (takeWhile ( $\lambda$ z. z  $\neq$  k) ks) = x"
  <proof>

```

```

lemma local_env_inited_LT:
  "[[ snd (local_env G C (mn, pTs) pns lvars) vname = Some T;
  length pns = length pTs; distinct pns; unique lvars ]
   $\Rightarrow$  (inited_LT C pTs lvars ! index (pns, lvars, blk, res) vname) = OK T"
  <proof>

```

```

lemma inited_LT_at_index_no_err:
  "i < length (inited_LT C pTs lvars)  $\Rightarrow$  inited_LT C pTs lvars ! i  $\neq$  Err"
  <proof>

```

```

lemma sup_loc_update_index: "
  [ G ⊢ T ≼ T'; is_type G T'; length pns = length pTs; distinct pns; unique lvars;
    snd (local_env G C (mn, pTs) pns lvars) vname = Some T' ]
  ⇒
  comp G ⊢ (inited_LT C pTs lvars) [index (pns, lvars, blk, res) vname := OK T] <=1
    inited_LT C pTs lvars"
  <proof>

```

#### 4.27.2 Preservation of ST and LT by compTpExpr / compTpStmt

```

lemma sttp_of_comb_nil [simp]: "sttp_of (comb_nil sttp) = sttp"
  <proof>

```

```

lemma mt_of_comb_nil [simp]: "mt_of (comb_nil sttp) = []"
  <proof>

```

```

lemma sttp_of_comb [simp]: "sttp_of ((f1 □ f2) sttp) = sttp_of (f2 (sttp_of (f1 sttp)))"
  <proof>

```

```

lemma mt_of_comb: "(mt_of ((f1 □ f2) sttp)) =
  (mt_of (f1 sttp)) @ (mt_of (f2 (sttp_of (f1 sttp))))"
  <proof>

```

```

lemma mt_of_comb_length [simp]: "[ n1 = length (mt_of (f1 sttp)); n1 ≤ n ]
  ⇒ (mt_of ((f1 □ f2) sttp) ! n) = (mt_of (f2 (sttp_of (f1 sttp))) ! (n - n1))"
  <proof>

```

```

lemma compTpExpr_Exprs_LT_ST: "
  [ jmb = (pns, lvars, blk, res);
    wf_prog wf_java_mdecl G;
    wf_java_mdecl G C ((mn, pTs), rT, jmb);
    E = local_env G C (mn, pTs) pns lvars ]
  ⇒
  (∀ ST LT T.
    E ⊢ ex :: T →
    is_inited_LT C pTs lvars LT →
    sttp_of (compTpExpr jmb G ex (ST, LT)) = (T # ST, LT))
  ∧
  (∀ ST LT Ts.
    E ⊢ exs [::] Ts →
    is_inited_LT C pTs lvars LT →
    sttp_of (compTpExprs jmb G exs (ST, LT)) = ((rev Ts) @ ST, LT))"
  <proof>

```

```

lemmas compTpExpr_LT_ST [rule_format (no_asm)] =
  compTpExpr_Exprs_LT_ST [THEN conjunct1]

```

```
lemmas compTpExprs_LT_ST [rule_format (no_asm)] =
  compTpExpr_Exprs_LT_ST [THEN conjunct2]
```

```
lemma compTpStmt_LT_ST [rule_format (no_asm)]: "
  [| jmb = (pns,lvars,blk,res);
   wf_prog wf_java_mdecl G;
   wf_java_mdecl G C ((mn, pTs), rT, jmb);
   E = (local_env G C (mn, pTs) pns lvars) |]
  ==> (∀ ST LT.
    E ⊢ s√ →
    (is_initiated_LT C pTs lvars LT)
    → sttp_of (compTpStmt jmb G s (ST, LT)) = (ST, LT))"

  <proof>
```

```
lemma compTpInit_LT_ST: "
  sttp_of (compTpInit jmb (vn,ty) (ST, LT)) = (ST, LT[(index jmb vn) := OK ty])"
  <proof>
```

```
lemma compTpInitLvars_LT_ST_aux [rule_format (no_asm)]:
  "∀ pre lvars_pre lvars0.
   jmb = (pns,lvars0,blk,res) ∧
   lvars0 = (lvars_pre @ lvars) ∧
   (length pns) + (length lvars_pre) + 1 = length pre ∧
   disjoint_varnames pns (lvars_pre @ lvars)
  →
  sttp_of (compTpInitLvars jmb lvars (ST, pre @ replicate (length lvars) Err))
    = (ST, pre @ map (Fun.comp OK snd) lvars)"
  <proof>
```

```
lemma compTpInitLvars_LT_ST:
  "[| jmb = (pns, lvars, blk, res); wf_java_mdecl G C ((mn, pTs), rT, jmb) |]
  ==> sttp_of (compTpInitLvars jmb lvars (ST, start_LT C pTs (length lvars)))
    = (ST, initiated_LT C pTs lvars)"
  <proof>
```

```
lemma max_of_list_elem: "x ∈ set xs ==> x ≤ (max_of_list xs)"
  <proof>
```

```
lemma max_of_list_sublist: "set xs ⊆ set ys
  ==> (max_of_list xs) ≤ (max_of_list ys)"
  <proof>
```

```
lemma max_of_list_append [simp]:
  "max_of_list (xs @ ys) = max (max_of_list xs) (max_of_list ys)"
  <proof>
```

**lemma** *app\_mono\_mxs*: "[ app i G mxs rT pc et s; mxs ≤ mxs' ]  
 ⇒ app i G mxs' rT pc et s"  
 ⟨proof⟩

**lemma** *err\_mono [simp]*: "A ⊆ B ⇒ err A ⊆ err B"  
 ⟨proof⟩

**lemma** *opt\_mono [simp]*: "A ⊆ B ⇒ opt A ⊆ opt B"  
 ⟨proof⟩

**lemma** *states\_mono*: "[ mxs ≤ mxs' ]  
 ⇒ states G mxs mxr ⊆ states G mxs' mxr"  
 ⟨proof⟩

**lemma** *check\_type\_mono*:  
 "[ check\_type G mxs mxr s; mxs ≤ mxs' ] ⇒ check\_type G mxs' mxr s"  
 ⟨proof⟩

**lemma** *wt\_instr\_prefix*: "  
 [ wt\_instr\_altern (bc ! pc) cG rT mt mxs mxr max\_pc et pc;  
 bc' = bc @ bc\_post; mt' = mt @ mt\_post;  
 mxs ≤ mxs'; max\_pc ≤ max\_pc';  
 pc < length bc; pc < length mt;  
 max\_pc = (length mt) ]  
 ⇒ wt\_instr\_altern (bc' ! pc) cG rT mt' mxs' mxr max\_pc' et pc"  
 ⟨proof⟩

**lemma** *pc\_succs\_shift*:  
 "pc' ∈ set (succs i (pc'' + n)) ⇒ ((pc' - n) ∈ set (succs i pc''))"  
 ⟨proof⟩

**lemma** *pc\_succs\_le*:  
 "[ pc' ∈ set (succs i (pc'' + n));  
 ∀ b. ((i = (Goto b) ∨ i = (Ifcmpeq b)) → 0 ≤ (int pc'' + b)) ]  
 ⇒ n ≤ pc'"  
 ⟨proof⟩

**definition** *offset\_xcentry* :: "[nat, exception\_entry] ⇒ exception\_entry" where

```

"offset_xcentry ==
  λ n (start_pc, end_pc, handler_pc, catch_type).
    (start_pc + n, end_pc + n, handler_pc + n, catch_type)"

```

```

definition offset_xctable :: "[nat, exception_table] ⇒ exception_table" where
  "offset_xctable n == (map (offset_xcentry n))"

```

```

lemma match_xcentry_offset [simp]: "
  match_exception_entry G cn (pc + n) (offset_xcentry n ee) =
  match_exception_entry G cn pc ee"
  <proof>

```

```

lemma match_xctable_offset: "
  (match_exception_table G cn (pc + n) (offset_xctable n et)) =
  (map_option (λ pc'. pc' + n) (match_exception_table G cn pc et))"
  <proof>

```

```

lemma match_offset [simp]: "
  match G cn (pc + n) (offset_xctable n et) = match G cn pc et"
  <proof>

```

```

lemma match_any_offset [simp]: "
  match_any G (pc + n) (offset_xctable n et) = match_any G pc et"
  <proof>

```

```

lemma app_mono_pc: "[ app i G mxs rT pc et s; pc' = pc + n ]
  ⇒ app i G mxs rT pc' (offset_xctable n et) s"
  <proof>

```

```

abbreviation (input)
  empty_et :: exception_table
  where "empty_et == []"

```

```

lemma xcpt_names_Nil [simp]: "(xcpt_names (i, G, pc, [])) = []"
  <proof>

```

```

lemma xcpt_eff_Nil [simp]: "(xcpt_eff i G pc s []) = []"
  <proof>

```

```

lemma app_jumps_lem: "[ app i cG mxs rT pc empty_et s; s=(Some st) ]
  ⇒ ∀ b. ((i = (Goto b) ∨ i=(Ifcmpeq b)) → 0 ≤ (int pc + b))"
  <proof>

```



```

lemma wt_instr_offset: "
  [ [  $\forall$  pc'' < length mt.
    wt_instr_altern ((bc@bc_post) ! pc'') cG rT (mt@mt_post) mxs mxr max_pc empty_et pc'';

    bc' = bc_pre @ bc @ bc_post; mt' = mt_pre @ mt @ mt_post;
    length bc_pre = length mt_pre; length bc = length mt;
    length mt_pre  $\leq$  pc; pc < length (mt_pre @ mt);
    mxs  $\leq$  mxs'; max_pc + length mt_pre  $\leq$  max_pc' ] ]
 $\Rightarrow$  wt_instr_altern (bc' ! pc) cG rT mt' mxs' mxr max_pc' empty_et pc"
  <proof>

```

```

definition start_sttp_resp_cons :: "[state_type  $\Rightarrow$  method_type  $\times$  state_type]  $\Rightarrow$  bool" where
  "start_sttp_resp_cons f ==
    ( $\forall$  sttp. let (mt', sttp') = (f sttp) in ( $\exists$  mt'_rest. mt' = Some sttp # mt'_rest))"

```

```

definition start_sttp_resp :: "[state_type  $\Rightarrow$  method_type  $\times$  state_type]  $\Rightarrow$  bool" where
  "start_sttp_resp f == (f = comb_nil)  $\vee$  (start_sttp_resp_cons f)"

```

```

lemma start_sttp_resp_comb_nil [simp]: "start_sttp_resp comb_nil"
  <proof>

```

```

lemma start_sttp_resp_cons_comb_cons [simp]: "start_sttp_resp_cons f
 $\Rightarrow$  start_sttp_resp_cons (f  $\square$  f')"
  <proof>

```

```

lemma start_sttp_resp_cons_comb_cons_r: "[ start_sttp_resp f; start_sttp_resp_cons f' ]
 $\Rightarrow$  start_sttp_resp_cons (f  $\square$  f')"
  <proof>

```

```

lemma start_sttp_resp_cons_comb [simp]: "start_sttp_resp_cons f
 $\Rightarrow$  start_sttp_resp (f  $\square$  f')"
  <proof>

```

```

lemma start_sttp_resp_comb: "[ start_sttp_resp f; start_sttp_resp f' ]
 $\Rightarrow$  start_sttp_resp (f  $\square$  f')"
  <proof>

```

```

lemma start_sttp_resp_cons_nochangeST [simp]: "start_sttp_resp_cons nochangeST"
  <proof>

```

```

lemma start_sttp_resp_cons_pushST [simp]: "start_sttp_resp_cons (pushST Ts)"
  <proof>

```

```

lemma start_sttp_resp_cons_dupST [simp]: "start_sttp_resp_cons dupST"
  <proof>

```

```

lemma start_sttp_resp_cons_dup_x1ST [simp]: "start_sttp_resp_cons dup_x1ST"
  <proof>

```

```

lemma start_sttp_resp_cons_popST [simp]: "start_sttp_resp_cons (popST n)"

```

*<proof>*

**lemma** *start\_sttp\_resp\_cons\_replST* [simp]: "start\_sttp\_resp\_cons (replST n tp)"  
*<proof>*

**lemma** *start\_sttp\_resp\_cons\_storeST* [simp]: "start\_sttp\_resp\_cons (storeST i tp)"  
*<proof>*

**lemma** *start\_sttp\_resp\_cons\_compTpExpr* [simp]: "start\_sttp\_resp\_cons (compTpExpr jmb G ex)"  
*<proof>*

**lemma** *start\_sttp\_resp\_cons\_compTpInit* [simp]: "start\_sttp\_resp\_cons (compTpInit jmb lv)"  
*<proof>*

**lemma** *start\_sttp\_resp\_nochangeST* [simp]: "start\_sttp\_resp nochangeST"  
*<proof>*

**lemma** *start\_sttp\_resp\_pushST* [simp]: "start\_sttp\_resp (pushST Ts)"  
*<proof>*

**lemma** *start\_sttp\_resp\_dupST* [simp]: "start\_sttp\_resp dupST"  
*<proof>*

**lemma** *start\_sttp\_resp\_dup\_x1ST* [simp]: "start\_sttp\_resp dup\_x1ST"  
*<proof>*

**lemma** *start\_sttp\_resp\_popST* [simp]: "start\_sttp\_resp (popST n)"  
*<proof>*

**lemma** *start\_sttp\_resp\_replST* [simp]: "start\_sttp\_resp (replST n tp)"  
*<proof>*

**lemma** *start\_sttp\_resp\_storeST* [simp]: "start\_sttp\_resp (storeST i tp)"  
*<proof>*

**lemma** *start\_sttp\_resp\_compTpExpr* [simp]: "start\_sttp\_resp (compTpExpr jmb G ex)"  
*<proof>*

**lemma** *start\_sttp\_resp\_compTpExprs* [simp]: "start\_sttp\_resp (compTpExprs jmb G exs)"  
*<proof>*

**lemma** *start\_sttp\_resp\_compTpStmt* [simp]: "start\_sttp\_resp (compTpStmt jmb G s)"  
*<proof>*

**lemma** *start\_sttp\_resp\_compTpInitLvars* [simp]: "start\_sttp\_resp (compTpInitLvars jmb lvars)"  
*<proof>*

### 4.27.3 length of compExpr/ compTpExprs

**lemma** *length\_comb* [simp]: "length (mt\_of ((f1  $\square$  f2) sttp)) =  
length (mt\_of (f1 sttp)) + length (mt\_of (f2 (sttp\_of (f1 sttp))))"  
*<proof>*

**lemma** length\_comb\_nil [simp]: "length (mt\_of (comb\_nil sttp)) = 0"  
 ⟨proof⟩

**lemma** length\_nochangeST [simp]: "length (mt\_of (nochangeST sttp)) = 1"  
 ⟨proof⟩

**lemma** length\_pushST [simp]: "length (mt\_of (pushST Ts sttp)) = 1"  
 ⟨proof⟩

**lemma** length\_dupST [simp]: "length (mt\_of (dupST sttp)) = 1"  
 ⟨proof⟩

**lemma** length\_dup\_x1ST [simp]: "length (mt\_of (dup\_x1ST sttp)) = 1"  
 ⟨proof⟩

**lemma** length\_popST [simp]: "length (mt\_of (popST n sttp)) = 1"  
 ⟨proof⟩

**lemma** length\_replST [simp]: "length (mt\_of (replST n tp sttp)) = 1"  
 ⟨proof⟩

**lemma** length\_storeST [simp]: "length (mt\_of (storeST i tp sttp)) = 1"  
 ⟨proof⟩

**lemma** length\_compTpExpr\_Exprs [rule\_format]: "  
 (∀ sttp. (length (mt\_of (compTpExpr jmb G ex sttp)) = length (compExpr jmb ex)))  
 ∧ (∀ sttp. (length (mt\_of (compTpExprs jmb G exs sttp)) = length (compExprs jmb exs)))"  
 ⟨proof⟩

**lemma** length\_compTpExpr: "length (mt\_of (compTpExpr jmb G ex sttp)) = length (compExpr jmb ex)"  
 ⟨proof⟩

**lemma** length\_compTpExprs: "length (mt\_of (compTpExprs jmb G exs sttp)) = length (compExprs jmb exs)"  
 ⟨proof⟩

**lemma** length\_compTpStmt [rule\_format]: "  
 (∀ sttp. (length (mt\_of (compTpStmt jmb G s sttp)) = length (compStmt jmb s)))"  
 ⟨proof⟩

**lemma** length\_compTpInit: "length (mt\_of (compTpInit jmb lv sttp)) = length (compInit jmb lv)"  
 ⟨proof⟩

**lemma** length\_compTpInitLvars [rule\_format]: "  
 "∀ sttp. length (mt\_of (compTpInitLvars jmb lvars sttp)) = length (compInitLvars jmb lvars)"  
 ⟨proof⟩

#### 4.27.4 Correspondence bytecode - method types

**abbreviation** (input)

$ST\_of :: "state\_type \Rightarrow opstack\_type"$   
 where  $"ST\_of == fst"$

**abbreviation** (input)

$LT\_of :: "state\_type \Rightarrow locvars\_type"$   
 where  $"LT\_of == snd"$

**lemma** *states\_lower*:

$"[ OK (Some (ST, LT)) \in states\ cG\ mxs\ mxr; length\ ST \leq mxs ]$   
 $\implies OK (Some (ST, LT)) \in states\ cG (length\ ST)\ mxr"$   
 $\langle proof \rangle$

**lemma** *check\_type\_lower*:

$"[ check\_type\ cG\ mxs\ mxr (OK (Some (ST, LT))); length\ ST \leq mxs ]$   
 $\implies check\_type\ cG (length\ ST)\ mxr (OK (Some (ST, LT)))"$   
 $\langle proof \rangle$

**definition** *bc\_mt\_corresp* :: "

$[bytecode, state\_type \Rightarrow method\_type \times state\_type, state\_type, jvm\_prog, ty, nat, p\_count]$   
 $\Rightarrow bool"$  where

$"bc\_mt\_corresp\ bc\ f\ sttp0\ cG\ rT\ mxr\ idx ==$   
 $let\ (mt, sttp) = f\ sttp0\ in$   
 $(length\ bc = length\ mt \wedge$   
 $((check\_type\ cG (length (ST\_of\ sttp0))\ mxr (OK (Some\ sttp0))) \longrightarrow$   
 $(\forall\ mxs.$   
 $mxs = max\_ssize\ (mt@[Some\ sttp]) \longrightarrow$   
 $(\forall\ pc. pc < idx \longrightarrow$   
 $wt\_instr\_altern\ (bc ! pc)\ cG\ rT\ (mt@[Some\ sttp])\ mxs\ mxr\ (length\ mt + 1)\ empty\_et$   
 $pc)$   
 $\wedge$   
 $check\_type\ cG\ mxs\ mxr (OK ((mt@[Some\ sttp]) ! idx)))))"$

**lemma** *bc\_mt\_corresp\_comb*:

$"[ bc' = (bc1@bc2); l' = (length\ bc');$   
 $bc\_mt\_corresp\ bc1\ f1\ sttp0\ cG\ rT\ mxr (length\ bc1);$   
 $bc\_mt\_corresp\ bc2\ f2 (sttp\_of (f1\ sttp0))\ cG\ rT\ mxr (length\ bc2);$   
 $start\_sttp\_resp\ f2 ]$   
 $\implies bc\_mt\_corresp\ bc' (f1 \sqcap f2)\ sttp0\ cG\ rT\ mxr\ l'"$   
 $\langle proof \rangle$

**lemma** *bc\_mt\_corresp\_zero [simp]*:

$"[ length (mt\_of (f\ sttp)) = length\ bc; start\_sttp\_resp\ f ]$   
 $\implies bc\_mt\_corresp\ bc\ f\ sttp\ cG\ rT\ mxr\ 0"$   
 $\langle proof \rangle$

**definition** `mt_sttp_flatten` :: "method\_type × state\_type ⇒ method\_type" where  
 "mt\_sttp\_flatten mt\_sttp == (mt\_of mt\_sttp) @ [Some (sttp\_of mt\_sttp)]"

**lemma** `mt_sttp_flatten_length` [simp]: "n = (length (mt\_of (f sttp)))  
 ⇒ (mt\_sttp\_flatten (f sttp)) ! n = Some (sttp\_of (f sttp))"  
 ⟨proof⟩

**lemma** `mt_sttp_flatten_comb`: "(mt\_sttp\_flatten ((f1 □ f2) sttp)) =  
 (mt\_of (f1 sttp)) @ (mt\_sttp\_flatten (f2 (sttp\_of (f1 sttp))))"  
 ⟨proof⟩

**lemma** `mt_sttp_flatten_comb_length` [simp]: "⌊ n1 = length (mt\_of (f1 sttp)); n1 ≤ n ⌋  
 ⇒ (mt\_sttp\_flatten ((f1 □ f2) sttp) ! n) = (mt\_sttp\_flatten (f2 (sttp\_of (f1 sttp))))  
 ! (n - n1))"  
 ⟨proof⟩

**lemma** `mt_sttp_flatten_comb_zero` [simp]:  
 "start\_sttp\_resp f ⇒ (mt\_sttp\_flatten (f sttp)) ! 0 = Some sttp"  
 ⟨proof⟩

**lemma** `int_outside_right`: "0 ≤ (m::int) ⇒ m + (int n) = int ((nat m) + n)"  
 ⟨proof⟩

**lemma** `int_outside_left`: "0 ≤ (m::int) ⇒ (int n) + m = int (n + (nat m))"  
 ⟨proof⟩

**lemma** `less_Suc` [simp] : "n ≤ k ⇒ (k < Suc n) = (k = n)"  
 ⟨proof⟩

**lemmas** `check_type_simps` = `check_type_def` `states_def` `JVMType.sl_def`  
`Product.esl_def` `stk_esl_def` `reg_sl_def` `upto_esl_def` `Listn.sl_def` `Err.sl_def`  
`JType.esl_def` `Err.esl_def` `Err.le_def` `Listn.le_def` `Product.le_def` `Product.sup_def` `Err.sup_def`  
`Opt.esl_def` `Listn.sup_def`

**lemma** `check_type_push`:  
 "⌊ is\_class cG cname; check\_type cG (length ST) mxr (OK (Some (ST, LT))) ⌋  
 ⇒ check\_type cG (Suc (length ST)) mxr (OK (Some (Class cname # ST, LT)))"  
 ⟨proof⟩

**lemma** `bc_mt_corresp_New`: "⌊ is\_class cG cname ⌋  
 ⇒ bc\_mt\_corresp [New cname] (pushST [Class cname]) (ST, LT) cG rT mxr (Suc 0)"

*<proof>*

**lemma** *bc\_mt\_corresp\_Pop*: "  
 $bc\_mt\_corresp \ [Pop] \ (popST \ (Suc \ 0)) \ (T \ \# \ ST, \ LT) \ cG \ rT \ mxr \ (Suc \ 0)''$   
*<proof>*

**lemma** *bc\_mt\_corresp\_Checkcast*: " $\llbracket is\_class \ cG \ cname; \ sttp = (ST, \ LT);$   
 $(\exists \ rT \ STo. \ ST = RefT \ rT \ \# \ STo) \ \rrbracket$   
 $\implies bc\_mt\_corresp \ [Checkcast \ cname] \ (replST \ (Suc \ 0) \ (Class \ cname)) \ sttp \ cG \ rT \ mxr \ (Suc \ 0)''$   
*<proof>*

**lemma** *bc\_mt\_corresp\_LitPush*: " $\llbracket \text{typeof} \ (\lambda v. \ None) \ val = Some \ T \ \rrbracket$   
 $\implies bc\_mt\_corresp \ [LitPush \ val] \ (pushST \ [T]) \ sttp \ cG \ rT \ mxr \ (Suc \ 0)''$   
*<proof>*

**lemma** *bc\_mt\_corresp\_LitPush\_CT*:  
 $\llbracket \text{typeof} \ (\lambda v. \ None) \ val = Some \ T \wedge \ cG \vdash T \preceq T'; \ is\_type \ cG \ T' \ \rrbracket$   
 $\implies bc\_mt\_corresp \ [LitPush \ val] \ (pushST \ [T']) \ sttp \ cG \ rT \ mxr \ (Suc \ 0)''$   
*<proof>*

**declare** *not\_Err\_eq* [iff del]

**lemma** *bc\_mt\_corresp\_Load*: " $\llbracket i < length \ LT; \ LT \ ! \ i \neq Err; \ mxr = length \ LT \ \rrbracket$   
 $\implies bc\_mt\_corresp \ [Load \ i]$   
 $(\lambda (ST, \ LT). \ pushST \ [ok\_val \ (LT \ ! \ i)] \ (ST, \ LT)) \ (ST, \ LT) \ cG \ rT \ mxr \ (Suc \ 0)''$   
*<proof>*

**lemma** *bc\_mt\_corresp\_Store\_init*:  
 $i < length \ LT \implies bc\_mt\_corresp \ [Store \ i] \ (storeST \ i \ T) \ (T \ \# \ ST, \ LT) \ cG \ rT \ mxr \ (Suc \ 0)''$   
*<proof>*

**lemma** *bc\_mt\_corresp\_Store*:  
 $\llbracket i < length \ LT; \ cG \vdash LT[i := OK \ T] \leq LT \ \rrbracket$   
 $\implies bc\_mt\_corresp \ [Store \ i] \ (popST \ (Suc \ 0)) \ (T \ \# \ ST, \ LT) \ cG \ rT \ mxr \ (Suc \ 0)''$   
*<proof>*

**lemma** *bc\_mt\_corresp\_Dup*: "  
 $bc\_mt\_corresp \ [Dup] \ dupST \ (T \ \# \ ST, \ LT) \ cG \ rT \ mxr \ (Suc \ 0)''$   
*<proof>*

**lemma** *bc\_mt\_corresp\_Dup\_x1*: "  
 $bc\_mt\_corresp \ [Dup\_x1] \ dup\_x1ST \ (T1 \ \# \ T2 \ \# \ ST, \ LT) \ cG \ rT \ mxr \ (Suc \ 0)''$   
*<proof>*

**lemma** *bc\_mt\_corresp\_IAdd*: "

```
bc_mt_corresp [IAdd] (replST 2 (PrimT Integer))
  (PrimT Integer # PrimT Integer # ST, LT) cG rT mxr (Suc 0)"
⟨proof⟩
```

```
lemma bc_mt_corresp_Getfield: "[ wf_prog wf_mb G;
  field (G, C) vname = Some (cname, T); is_class G C ]
⇒ bc_mt_corresp [Getfield vname cname]
  (replST (Suc 0) (snd (the (field (G, cname) vname))))
  (Class C # ST, LT) (comp G) rT mxr (Suc 0)"
⟨proof⟩
```

```
lemma bc_mt_corresp_Putfield: "[ wf_prog wf_mb G;
  field (G, C) vname = Some (cname, Ta); G ⊢ T ≼ Ta; is_class G C ]
⇒ bc_mt_corresp [Putfield vname cname] (popST 2) (T # Class C # T # ST, LT)
  (comp G) rT mxr (Suc 0)"
⟨proof⟩
```

```
lemma Call_app:
  "[ wf_prog wf_mb G; is_class G cname;
    STs = rev pTsa @ Class cname # ST;
    max_spec G cname (mname, pTsa) = {((md, T), pTs')} ]
⇒ app (Invoke cname mname pTs') (comp G) (length (T # ST)) rT 0 empty_et (Some (STs,
  LTs))"
⟨proof⟩
```

```
lemma bc_mt_corresp_Invoke:
  "[ wf_prog wf_mb G;
    max_spec G cname (mname, pTsa) = {((md, T), fpTs)};
    is_class G cname ]
⇒ bc_mt_corresp [Invoke cname mname fpTs] (replST (Suc (length pTsa)) T)
  (rev pTsa @ Class cname # ST, LT) (comp G) rT mxr (Suc 0)"
⟨proof⟩
```

```
lemma wt_instr_Ifcmpeq: "[ Suc pc < max_pc;
  0 ≤ (int pc + i); nat (int pc + i) < max_pc;
  (mt_sttp_flatten f ! pc = Some (ts#ts'#ST,LT)) ∧
  ((∃ p. ts = PrimT p ∧ ts' = PrimT p) ∨ (∃ r r'. ts = RefT r ∧ ts' = RefT r'));
  mt_sttp_flatten f ! Suc pc = Some (ST,LT);
  mt_sttp_flatten f ! nat (int pc + i) = Some (ST,LT);
  check_type (TranslComp.comp G) mxs mxr (OK (Some (ts # ts' # ST, LT))) ]
⇒ wt_instr_altern (Ifcmpeq i) (comp G) rT (mt_sttp_flatten f) mxs mxr max_pc empty_et
  pc"
⟨proof⟩
```

```
lemma wt_instr_Goto: "[ 0 ≤ (int pc + i); nat (int pc + i) < max_pc;
  mt_sttp_flatten f ! nat (int pc + i) = (mt_sttp_flatten f ! pc);
  check_type (TranslComp.comp G) mxs mxr (OK (mt_sttp_flatten f ! pc)) ]
⇒ wt_instr_altern (Goto i) (comp G) rT (mt_sttp_flatten f) mxs mxr max_pc empty_et
  pc"
```

*<proof>*

```

lemma bc_mt_corresp_comb_inside: "
  [
    bc_mt_corresp bc' f' sttp0 cG rT mxr l1;
    bc' = (bc1@bc2@bc3); f' = (f1 □ f2 □ f3);
    l1 = (length bc1); l12 = (length (bc1@bc2));
    bc_mt_corresp bc2 f2 (sttp_of (f1 sttp0)) cG rT mxr (length bc2);
    length bc1 = length (mt_of (f1 sttp0));
    start_sttp_resp f2; start_sttp_resp f3]
  ⇒ bc_mt_corresp bc' f' sttp0 cG rT mxr l12"
<proof>

```

```

definition contracting :: "(state_type ⇒ method_type × state_type) ⇒ bool" where
  "contracting f == (∀ ST LT.
    let (ST', LT') = sttp_of (f (ST, LT))
    in (length ST' ≤ length ST ∧ set ST' ⊆ set ST ∧
        length LT' = length LT ∧ set LT' ⊆ set LT))"

```

```

lemma set_drop_Suc [rule_format]: "∀ xs. set (drop (Suc n) xs) ⊆ set (drop n xs)"
<proof>

```

```

lemma set_drop_le [rule_format,simp]: "∀ n xs. n ≤ m → set (drop m xs) ⊆ set (drop n xs)"
<proof>

```

```

declare set_drop_subset [simp]

```

```

lemma contracting_popST [simp]: "contracting (popST n)"
<proof>

```

```

lemma contracting_nochangeST [simp]: "contracting nochangeST"
<proof>

```

```

lemma check_type_contracting: "[ check_type cG mxs mxr (OK (Some sttp)); contracting f ]
  ⇒ check_type cG mxs mxr (OK (Some (sttp_of (f sttp))))"
<proof>

```



```

lemma bc_mt_corresp_comb_wt_instr: "
  [[ bc_mt_corresp bc' f' sttp0 cG rT mxr l1;
    bc' = (bc1@[inst]@bc3); f' = (f1 □ f2 □ f3);
    l1 = (length bc1);
    length bc1 = length (mt_of (f1 sttp0));
    length (mt_of (f2 (sttp_of (f1 sttp0)))) = 1;
    start_sttp_resp_cons f1; start_sttp_resp_cons f2; start_sttp_resp f3;

    check_type cG (max_ssize (mt_sttp_flatten (f' sttp0))) mxr
      (OK ((mt_sttp_flatten (f' sttp0)) ! (length bc1)))
  ] ]
  →
  wt_instr_altern inst cG rT
    (mt_sttp_flatten (f' sttp0))
    (max_ssize (mt_sttp_flatten (f' sttp0)))
    mxr
    (Suc (length bc'))
    empty_et
    (length bc1);
  contracting f2
] ]
⇒ bc_mt_corresp bc' f' sttp0 cG rT mxr (length (bc1@[inst]))"
  <proof>

```

```

lemma compTpExpr_LT_ST_rewr [simp]:
  "[ wf_java_prog G; wf_java_mdecl G C ((mn, pTs), rT, (pns, lvars, blk, res));
    local_env G C (mn, pTs) pns lvars ⊢ ex :: T;
    is_initiated_LT C pTs lvars LT ] ]
  ⇒ sttp_of (compTpExpr (pns, lvars, blk, res) G ex (ST, LT)) = (T # ST, LT)"
  <proof>

```

```

lemma wt_method_compTpExpr_Exprs_corresp: "
  [[ jmb = (pns, lvars, blk, res);
    wf_prog wf_java_mdecl G;
    wf_java_mdecl G C ((mn, pTs), rT, jmb);
    E = (local_env G C (mn, pTs) pns lvars) ] ]
  ⇒
  (∀ ST LT T bc' f'.
    E ⊢ ex :: T →
    (is_initiated_LT C pTs lvars LT) →
    bc' = (compExpr jmb ex) →
    f' = (compTpExpr jmb G ex)
    → bc_mt_corresp bc' f' (ST, LT) (comp G) rT (length LT) (length bc'))
  ∧
  (∀ ST LT Ts.
    E ⊢ exs [::] Ts →
    (is_initiated_LT C pTs lvars LT)
    → bc_mt_corresp (compExprs jmb exs) (compTpExprs jmb G exs) (ST, LT) (comp G) rT (length
    LT) (length (compExprs jmb exs)))"
  <proof>

```

```

lemmas wt_method_compTpExpr_corresp [rule_format (no_asm)] =

```

`wt_method_compTpExpr_Exprs_corresp [THEN conjunct1]`

```

lemma wt_method_compTpStmt_corresp [rule_format (no_asm)]: "
  [| jmb = (pns,lvars,blk,res);
   wf_prog wf_java_mdecl G;
   wf_java_mdecl G C ((mn, pTs), rT, jmb);
   E = (local_env G C (mn, pTs) pns lvars)|]
  ==>
  (∀ ST LT T bc' f'.
   E ⊢ s√ →
   (is_initiated_LT C pTs lvars LT) →
   bc' = (compStmt jmb s) →
   f' = (compTpStmt jmb G s)
   → bc_mt_corresp bc' f' (ST, LT) (comp G) rT (length LT) (length bc'))"

  <proof>

```

```

lemma wt_method_compTpInit_corresp: "[| jmb = (pns,lvars,blk,res);
   wf_java_mdecl G C ((mn, pTs), rT, jmb); mxr = length LT;
   length LT = (length pns) + (length lvars) + 1; vn ∈ set (map fst lvars);
   bc = (compInit jmb (vn,ty)); f = (compTpInit jmb (vn,ty));
   is_type G ty |]
  ==> bc_mt_corresp bc f (ST, LT) (comp G) rT mxr (length bc)"

  <proof>

```

```

lemma wt_method_compTpInitLvars_corresp_aux [rule_format (no_asm)]: "
  ∀ lvars_pre lvars0 ST LT.
  jmb = (pns,lvars0,blk,res) ∧
  lvars0 = (lvars_pre @ lvars) ∧
  length LT = (length pns) + (length lvars0) + 1 ∧
  wf_java_mdecl G C ((mn, pTs), rT, jmb)
  → bc_mt_corresp (compInitLvars jmb lvars) (compTpInitLvars jmb lvars) (ST, LT) (comp
  G) rT
    (length LT) (length (compInitLvars jmb lvars))"

  <proof>

```

```

lemma wt_method_compTpInitLvars_corresp: "[| jmb = (pns,lvars,blk,res);
   wf_java_mdecl G C ((mn, pTs), rT, jmb);
   length LT = (length pns) + (length lvars) + 1; mxr = (length LT);
   bc = (compInitLvars jmb lvars); f = (compTpInitLvars jmb lvars) |]
  ==> bc_mt_corresp bc f (ST, LT) (comp G) rT mxr (length bc)"

```

$\langle proof \rangle$

```

lemma wt_method_comp_wo_return: "[ wf_prog wf_java_mdecl G;
  wf_java_mdecl G C ((mn, pTs), rT, jmb);
  bc = compInitLvars jmb lvars @ compStmnt jmb blk @ compExpr jmb res;
  jmb = (pns, lvars, blk, res);
  f = (compTpInitLvars jmb lvars □ compTpStmnt jmb G blk □ compTpExpr jmb G res);
  sttp = (start_ST, start_LT C pTs (length lvars));
  li = (length (inited_LT C pTs lvars))
]
⇒ bc_mt_corresp bc f sttp (comp G) rT li (length bc)"
 $\langle proof \rangle$ 

```

```

lemma check_type_start:
  "[ wf_mhead cG (mn, pTs) rT; is_class cG C ]
⇒ check_type cG (length start_ST) (Suc (length pTs + mxl))
  (OK (Some (start_ST, start_LT C pTs mxl)))"
 $\langle proof \rangle$ 

```

```

lemma wt_method_comp_aux:
  "[ bc' = bc @ [Return]; f' = (f □ nochangeST);
  bc_mt_corresp bc f sttp0 cG rT (1+length pTs+mxl) (length bc);
  start_sttp_resp_cons f';
  sttp0 = (start_ST, start_LT C pTs mxl);
  mxs = max_ssize (mt_of (f' sttp0));
  wf_mhead cG (mn, pTs) rT; is_class cG C;
  sttp_of (f sttp0) = (T # ST, LT);

  check_type cG mxs (1+length pTs+mxl) (OK (Some (T # ST, LT))) →
  wt_instr_altern Return cG rT (mt_of (f' sttp0)) mxs (1+length pTs+mxl)
  (Suc (length bc)) empty_et (length bc)
]
⇒ wt_method_altern cG C pTs rT mxs mxl bc' empty_et (mt_of (f' sttp0))"
 $\langle proof \rangle$ 

```

```

lemma wt_instr_Return: "[fst f ! pc = Some (T # ST, LT); (G ⊢ T ≤ rT); pc < max_pc;
  check_type (TranslComp.comp G) mxs mxr (OK (Some (T # ST, LT)))
]
⇒ wt_instr_altern Return (comp G) rT (mt_of f) mxs mxr max_pc empty_et pc"
 $\langle proof \rangle$ 

```

```

theorem wt_method_comp: "
  [| wf_java_prog G; (C, D, fds, mths) ∈ set G; jmdcl ∈ set mths;
    jmdcl = ((mn,pTs), rT, jmb);
    mt = (compTpMethod G C jmdcl);
    (mxs, mxl, bc, et) = mtd_mb (compMethod G C jmdcl) |]
  ⇒ wt_method (comp G) C pTs rT mxs mxl bc et mt"

```

⟨proof⟩

```

lemma comp_set_ms: "(C, D, fs, cms) ∈ set (comp G)
  ⇒ ∃ ms. (C, D, fs, ms) ∈ set G ∧ cms = map (compMethod G C) ms"
  ⟨proof⟩

```

#### 4.27.5 Main Theorem

```

theorem wt_prog_comp: "wf_java_prog G ⇒ wt_jvm_prog (comp G) (compTp G)"
  ⟨proof⟩

```

```

declare split_paired_All [simp add]
declare split_paired_Ex [simp add]

```

```

end
theory MicroJava
imports
  "J/JTypeSafe"
  "J/Example"
  "J/JListExample"
  "JVM/JVMListExample"
  "JVM/JVMDefensive"
  "BV/LBVJVM"
  "BV/BVNoTypeError"
  "BV/BVExample"
  "Comp/CorrComp"
  "Comp/CorrCompTp"
begin

end

```

# Bibliography

- [1] G. Klein and T. Nipkow. Verified lightweight bytecode verification. In S. Drossopoulou, S. Eisenbach, B. Jacobs, G. T. Leavens, P. Müller, and A. Poetzsch-Heffter, editors, *Formal Techniques for Java Programs*. Technical Report 269, 5/2000, Fernuniversität Hagen, 2000. ECOOP2000 Workshop proceedings available from <http://www.informatik.fernuni-hagen.de/pi5/publications.html>.
- [2] G. Klein and T. Nipow. Verified lightweight bytecode verification. *Concurrency and Computation: Practice and Experience*, 13(13):1133–1151, 2001. Invited contribution to special issue on Formal Techniques for Java.
- [3] T. Nipkow. Verified bytecode verifiers. In F. Honsell, editor, *Foundations of Software Science and Computation Structures (FOSSACS 2001)*, volume 2030, pages 347–363, 2001.
- [4] T. Nipkow, D. v. Oheimb, and C. Pusch.  $\mu$ Java: Embedding a programming language in a theorem prover. In F. L. Bauer and R. Steinbrüggen, editors, *Foundations of Secure Computation*, volume 175 of *NATO Science Series F: Computer and Systems Sciences*, pages 117–144. IOS Press, 2000.
- [5] D. von Oheimb. Axiomatic semantics for Java<sup>light</sup> in Isabelle/HOL. In S. Drossopoulou, S. Eisenbach, B. Jacobs, G. T. Leavens, P. Müller, and A. Poetzsch-Heffter, editors, *Formal Techniques for Java Programs*. Technical Report 269, 5/2000, Fernuniversität Hagen, 2000. ECOOP2000 Workshop proceedings available from <http://www.informatik.fernuni-hagen.de/pi5/publications.html>.